Solving linear programming problems using the graphical method
Example - designing a diet

A dietitian wants to design a breakfast menu for certain hospital patients. The menu is to include two items A and B. Suppose that each ounce of A provides 2 units of vitamin C and 2 units of iron and each ounce of B provides 1 unit of vitamin C and 2 units of iron. Suppose the cost of A is 4¢/ounce and the cost of B is 3¢/ounce. If the breakfast menu must provide at least 8 units of vitamin C and 10 units of iron, how many ounces of each item should be provided in order to meet the iron and vitamin C requirements for the least cost? What will this breakfast cost?
\[ x = \text{#oz. of } A \]
\[ y = \text{#oz. of } B \]

\begin{align*}
\text{vit. C:} & \quad 2x + y \geq 8 \\
\text{iron:} & \quad 2x + 2y \geq 10 \\
\text{x \geq 0, y \geq 0} & \quad x \geq 0, y \geq 0
\end{align*}

\text{Cost} = C = 4x + 3y
\[ x = \# \text{oz. of A} \]
\[ y = \# \text{oz. of B} \]

\[
\begin{align*}
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Cost = \[ C = 4x + 3y \]
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Cost = \[C = 4x + 3y\]
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- Vit. C: \[ 2x + y \geq 8 \]
- Iron: \[ 2x + 2y \geq 10 \]
  \[ x \geq 0, \ y \geq 0 \]

Cost: \[ C = 4x + 3y \]

The 3 blue lines are:
- \[ 48 = 4x + 3y \]
- \[ 36 = 4x + 3y \]
- \[ 24 = 4x + 3y \]

\[ x = \# \text{oz. of A} \]
\[ y = \# \text{oz. of B} \]
The cost will be minimized if the strategy followed is the one corresponding to this corner point.

Cost = $C = 4x + 3y$

The 3 blue lines are:
- $48 = 4x + 3y$
- $36 = 4x + 3y$
- $24 = 4x + 3y$

$x = \#\text{oz. of } A$
$y = \#\text{oz. of } B$
\[ x = \# \text{oz. of A} \]
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vit. C: \[ 2x + y \geq 8 \]
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\[ 2x + y = 8 \]
\[ 2x + 2y = 10 \]
\[ x = \text{#oz. of A} \]
\[ y = \text{#oz. of B} \]

\begin{align*}
\text{vit. C:} & \quad 2x + y \geq 8 \\
\text{iron:} & \quad 2x + 2y \geq 10 \\
& \quad x \geq 0, \ y \geq 0
\end{align*}

\[ 2x + y = 8 \]
\[ 2x + 2y = 10 \]

Solution: \( x=3, \ y=2 \)
\[ C = 4x + 3y = 18\$ \]
\[ x = \# \text{oz. of A} \]
\[ y = \# \text{oz. of B} \]

\[
\begin{array}{c|c|c}
\text{corner pt.} & C = 4x + 3y & \\
(0,8) & 24 \text{ cents} & \\
(5,0) & 20 \text{ cents} & \\
(3,2) & 18 \text{ cents} & \\
\end{array}
\]
Example - bicycle factories

A small business makes 3-speed and 10-speed bicycles at two different factories. Factory A produces 16 3-speed and 20 10-speed bikes in one day while factory B produces 12 3-speed and 20 10-speed bikes daily. It costs $1000/day to operate factory A and $800/day to operate factory B. An order for 96 3-speed bikes and 140 10-speed bikes has just arrived. How many days should each factory be operated in order to fill this order at a minimum cost? What is the minimum cost?

\[ x = \# \text{ days factory A is operated} \]
\[ y = \# \text{ days factory B is operated} \]
\[x = \# \text{ days factory A is operated}\]
\[y = \# \text{ days factory B is operated}\]
\[ x = \# \text{ days factory A is operated} \]
\[ y = \# \text{ days factory B is operated} \]

3-speed constraint: \[ 16x + 12y \geq 96 \]
x = # days factory A is operated
y = # days factory B is operated
3-speed constraint: 16x + 12y ≥ 96
10-speed constraint: 20x + 20y ≥ 140
x ≥ 0, y ≥ 0
\begin{align*}
x &= \# \text{ days factory } A \text{ is operated} \\
y &= \# \text{ days factory } B \text{ is operated} \\
3\text{-speed constraint: } &16x + 12y \geq 96 \\
10\text{-speed constraint: } &20x + 20y \geq 140 \\
x \geq 0, \ y \geq 0 \\
\text{Minimize: } &C = 1000x + 800y
\end{align*}
\[ x = \text{# days factory A is operated} \]
\[ y = \text{# days factory B is operated} \]

3-speed constraint: \[16x + 12y \geq 96\]
10-speed constraint: \[20x + 20y \geq 140\]
\[ x \geq 0, \ y \geq 0 \]

Minimize: \[C = 1000x + 800y\]
3-speed constraint: \( 16x + 12y \geq 96 \)
10-speed constraint: \( 20x + 20y \geq 140 \)
\( x \geq 0, y \geq 0 \)

Minimize: \( C = 1000x + 800y \)
x = # days factory A is operated
y = # days factory B is operated

3-speed constraint: $16x + 12y \geq 96$
10-speed constraint: $20x + 20y \geq 140$

$x \geq 0, y \geq 0$

Minimize: $C = 1000x + 800y$
x = # days factory A is operated
y = # days factory B is operated

3-speed constraint: 16x + 12y ≥ 96
10-speed constraint: 20x + 20y ≥ 140

x ≥ 0, y ≥ 0

Minimize: C = 1000x + 800y

corner pts C = 1000x + 800y

3-speed

10-speed
3-speed constraint: $16x + 12y \geq 96$
10-speed constraint: $20x + 20y \geq 140$

$x \geq 0, \ y \geq 0$

Minimize: $C = 1000x + 800y$

Corner pts:
- 3-speed: $(0,8)\ \ \ C = 10000 + 6400$
- 10-speed: $(8,0)\ \ \ C = 10000 + 0$

$C = 10000 + 6400$
$C = 10000 + 0$

$C = 6400$
$C = 10000$
3-speed constraint: $16x + 12y \geq 96$
10-speed constraint: $20x + 20y \geq 140$

Minimize: $C = 1000x + 800y$

$x \geq 0, y \geq 0$

Corner points:
- $(0,8): C = 6400$
- $(7,0): C = 7000$

$x = \# \text{ days factory } A \text{ is operated}$
$y = \# \text{ days factory } B \text{ is operated}$

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\[ x = \text{\# days factory A is operated} \]
\[ y = \text{\# days factory B is operated} \]

3-speed constraint: \( 16x + 12y \geq 96 \)

10-speed constraint: \( 20x + 20y \geq 140 \)

\[ x \geq 0, \ y \geq 0 \]

Minimize: \( C = 1000x + 800y \)

\[
\begin{array}{c|c|c}
\text{corner pts} & C = 1000x + 800y & \\
(0,8) & $6400 & \\
(7,0) & $7000 & \\
(3,4) & $6200 & \\
\end{array}
\]
Example - ski manufacturing

Michigan Polar Products makes downhill and cross-country skis. A pair of downhill skis requires 2 man-hours for cutting, 1 man-hour for shaping and 3 man-hours for finishing while a pair of cross-country skis requires 2 man-hours for cutting, 2 man-hours for shaping and 1 man-hour for finishing. Each day the company has available 140 man-hours for cutting, 120 man-hours for shaping and 150 man-hours for finishing. How many pairs of each type of ski should the company manufacture each day in order to maximize profit if a pair of downhill skis yields a profit of $10 and a pair of cross-country skis yields a profit of $8?
\[ x = \# \text{ pairs of downhill skis} \]
\[ y = \# \text{ pairs of cross country skis} \]

- **cutting:** \[ 2x + 2y \leq 140 \]
- **shaping:** \[ x + 2y \leq 120 \]
- **finishing:** \[ 3x + y \leq 150 \]
  \[ x \geq 0, \quad y \geq 0 \]

\[ P = 10x + 8y \]
x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: \[2x + 2y \leq 140\]
shaping: \[x + 2y \leq 120\]
finishing: \[3x + y \leq 150\]

\[x \geq 0, y \geq 0\]

\[P = 10x + 8y\]
\[x = \# \text{ pairs of downhill skis}\]
\[y = \# \text{ pairs of cross country skis}\]

- **cutting:** \[2x + 2y \leq 140\]
- **shaping:** \[x + 2y \leq 120\]
- **finishing:** \[3x + y \leq 150\]

\[x \geq 0, y \geq 0\]

\[P = 10x + 8y\]
x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: \( 2x + 2y \leq 140 \)
shaping: \( x + 2y \leq 120 \)
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\( x \geq 0, y \geq 0 \)

\( P = 10x + 8y \)
x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: $2x + 2y \leq 140$
shaping: $x + 2y \leq 120$
finishing: $3x + y \leq 150$

$x \geq 0, y \geq 0$

$P = 10x + 8y$
\( x = \# \) pairs of downhill skis
\( y = \# \) pairs of cross country skis

\[
\begin{align*}
\text{cutting:} & \quad 2x + 2y \leq 140 \\
\text{shaping:} & \quad x + 2y \leq 120 \\
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& \quad x \geq 0 \text{, } y \geq 0
\end{align*}
\]

\( P = 10x + 8y \)
x = # pairs of downhill skis
y = # pairs of cross country skis

- cutting: \[2x + 2y \leq 140\]
- shaping: \[x + 2y \leq 120\]
- finishing: \[3x + y \leq 150\]

\[x \geq 0, \ y \geq 0\]

\[P = 10x + 8y\]

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<tr>
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x = # pairs of downhill skis
y = # pairs of cross country skis

- cutting: \[2x + 2y \leq 140\]
- shaping: \[x + 2y \leq 120\]
- finishing: \[3x + y \leq 150\]
- \[x \geq 0, y \geq 0\]

Make 40 pairs of downhill skis and 30 pairs of cross country skis for a profit of $640

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