Problem 9.1  In Active Example 9.1, suppose that the coefficient of static friction between the 180-N crate and the ramp is \( \mu_s = 0.3 \). What is the magnitude of the smallest horizontal force the rope must exert on the crate to prevent it from sliding down the ramp?

Solution:  The free-body diagram is shown. Assume that the crate is on the verge of slipping down the ramp. Then the friction force is \( f = \mu_s N \) and points up the ramp. The equilibrium equations are

\[
\sum F_x : N - \mu_s N + T \cos 20^\circ - W \sin 20^\circ = 0, \\
\sum F_y : N - T \sin 20^\circ - W \cos 20^\circ = 0.
\]

Setting \( W = 180 \text{ N} \) and \( \mu_s = 0.3 \) and solving yields

\[ N = 173 \text{ N}, T = 10.4 \text{ N}. \]

Problem 9.2  A person places a 10 N book on a table that is tilted at 5° relative to the horizontal. She finds that if she exerts a very small force on the book as shown, the book remains in equilibrium, but if she removes the force, the book slides down the table. What force would she need to exert on the book (in the direction parallel to the table) to cause it to slide up the table?

Solution:  If the person can hold the book in equilibrium with a very small force, but the book slips when she removes the force, the maximum friction force must be nearly equal to the value necessary to maintain the book in equilibrium. Assume that the book is in equilibrium and \( f = \mu_s N \) (Fig. a). The equilibrium equations are

\[
\sum F_x : f - (10 \text{ N}) \sin 15^\circ = 0, \\
\sum F_y : N - (10 \text{ N}) \cos 15^\circ = 0.
\]

The coefficient of friction is then

\[ \mu_s = \frac{f}{N} = \tan 15^\circ = 0.268. \]

Now assume that the person exerts a force \( F \) on the book that is parallel to the table and slip up the table is impending (Fig. b). Then the friction force \( f = \mu_s N \) opposes the impending motion and the equilibrium equations are

\[
\sum F_x : F - f - (10 \text{ N}) \sin 15^\circ = 0, \\
\sum F_y : N - (10 \text{ N}) \cos 15^\circ = 0.
\]

Solving yields \( F = 5.176 \text{ N} \).
Problem 9.3  A student pushes a 200-N box of books across the floor. The coefficient of kinetic friction between the carpet and the box is $\mu_k = 0.15$.

(a) If he exerts the force $F$ at angle $\alpha = 25^\circ$, what is the magnitude of the force he must exert to slide the box across the floor?

(b) If he bends his knees more and exerts the force $F$ at angle $\alpha = 10^\circ$, what is the magnitude of the force he must exert to slide the box?

Solution:

$\sum F_x : F \cos \alpha - f = 0$

$\sum F_y : N - 200 \text{ N} - F \sin \alpha = 0$

$f = 0.15 \text{ N}$

(a) $\alpha = 25^\circ \Rightarrow F = 35.6 \text{ N}$

(b) $\alpha = 10^\circ \Rightarrow F = 31.3 \text{ N}$

Problem 9.4  The 15 kN car is parked on a sloped street. The brakes are applied to both its front and rear wheels.

(a) If the coefficient of static friction between the car’s tires and the road is $\mu_s = 0.8$, what is the steepest slope (in degrees relative to the horizontal) on which the car could remain in equilibrium?

(b) If the street were icy and the coefficient of static friction between the car’s tires and the road was $\mu_s = 0.2$, what is the steepest slope on which the car could remain in equilibrium?

Solution: Let $\alpha$ be the slope of the street in degrees. The equilibrium equations and impending slip friction equation are

$\sum F_x : W \sin \alpha - f = 0.$

$\sum F_x : N - W \cos \alpha = 0.$

$f = \mu_s N$

Solving, we find that

$f = W \sin \alpha, N = W \cos \alpha, \mu_s = \tan \alpha.$

$\alpha = \tan^{-1}(\mu_s).$

(a) $\alpha = \tan^{-1}(0.8) = 38.7^\circ.$

(b) $\alpha = \tan^{-1}(0.2) = 11.3^\circ.$

(a) $\alpha = 38.7^\circ$, (b) $\alpha = 11.3^\circ$. 
Problem 9.5 The truck’s winch exerts a horizontal force on the 200-kg crate in an effort to pull it down the ramp. The coefficient of static friction between the crate and the ramp is \( \mu_s = 0.6 \).

(a) If the winch exerts a 200-N horizontal force on the crate, what is the magnitude of the friction force exerted on the crate by the ramp?

(b) What is the magnitude of the horizontal force the winch must exert on the crate to cause it to start moving down the ramp?

Solution: Assume the crate doesn’t slip.

\[ \sum \mathbf{F}_N : N - 1962 \, \text{N} \cos 20^\circ + 200 \, \text{N} \sin 20^\circ = 0 \]

\[ \sum \mathbf{F}_f : f - 1962 \, \text{N} \sin 20^\circ - 200 \, \text{N} \cos 20^\circ = 0 \]

\( f_{\text{max}} = 0.6 \, \text{N} \)

(a) Solving

\[ N = 1775 \, \text{N}, \quad f = 859 \, \text{N}, \quad f_{\text{max}} = 1065 \, \text{N} \]

Since \( f < f_{\text{max}} \)

\[ f = 859 \, \text{N} \]

(b) \[ \sum \mathbf{F}_N : N - 1962 \, \text{N} \cos 20^\circ + F \sin 20^\circ = 0 \]

\[ \sum \mathbf{F}_f : f - 1962 \, \text{N} \sin 20^\circ - F \cos 20^\circ = 0 \quad \Rightarrow \quad F = 380 \, \text{N} \]

\( f = 0.6 \, \text{N} \)
Problem 9.6  The device shown is designed to position pieces of luggage on a ramp. It exerts a force parallel to the ramp. The suitcase weighs 200 N. The coefficients of friction between the suitcase and ramp are $\mu_s = 0.20$ and $\mu_k = 0.18$.

(a) Will the suitcase remain stationary on the ramp when the device exerts no force on it?
(b) What force must the device exert to push the suitcase up the ramp at a constant speed?

Solution:

(a) Assume that the suitcase is in equilibrium with no external force exerted on it (Fig. a). From the equilibrium equations

\[ \Sigma F_x : f - W \sin 20^\circ = 0, \]
\[ \Sigma F_y : N - W \cos 20^\circ = 0, \]

we obtain

\[ f = W \sin 20^\circ = 68.4 \text{ N}, \]
\[ N = W \cos 20^\circ = 187.9 \text{ N}. \]

The maximum friction force

\[ f_{\text{max}} = \mu_s N = (0.2)(187.9 \text{ N}) = 37.6 \text{ N} \]

is less than the friction force necessary for equilibrium, so the suitcase will not remain in equilibrium with no force exerted on it.

(b) Now assume that the device exerts a force $F$ on the suitcase and is pushing it up the ramp at a constant speed (Fig. b). Then the friction force $f = \mu_k N$ opposes the motion and the equilibrium equations are

\[ \Sigma F_x : F - \mu_k N - W \sin 20^\circ = 0, \]
\[ \Sigma F_y : N - W \cos 20^\circ = 0. \]

Solving yields $N = 187.9 \text{ N}, F = 102.2 \text{ N}$. (a) No, (b) 102.2 N.
Problem 9.7  The coefficient of static friction between the 50-kg crate and the ramp is \( \mu_s = 0.35 \). The unstretched length of the spring is 800 mm, and the spring constant is \( k = 660 \, \text{N/m} \).

What is the minimum value of \( x \) at which the crate can remain stationary on the ramp?

Solution:

\[
F_s = (660 \, \text{N/m})(0.8 \, \text{m} - x)
\]

\[
\sum F_x : F_s - 490.5 \, \text{N} \sin 50^\circ + f = 0
\]

\[
\sum F_y : N - 490.5 \, \text{N} \cos 50^\circ = 0
\]

\[
f = 0.35 \, \text{N}
\]

Solving: \( x = 0.398 \, \text{m} = 398 \, \text{mm} \)

Problem 9.8  The coefficient of kinetic friction between the 40-kg crate and the slanting floor is \( \mu_k = 0.3 \). If the angle \( \alpha = 20^\circ \), what tension must the person exert on the rope to move the crate at constant speed?

Solution:

\[
\alpha = 20^\circ, \mu_k = 0.3
\]

\[
\sum F_x : T \cos \alpha - f - 392.4 \, \text{N} \sin 10^\circ = 0
\]

\[
\sum F_y : T \sin \alpha + N - 392.4 \, \text{N} \cos 10^\circ = 0
\]

\[
f = 0.3 \, \text{N}
\]

Solving: \( T = 177 \, \text{N} \)
Problem 9.9  In Problem 9.8, for what angle $\alpha$ is the tension necessary to move the crate at constant speed a minimum? What is the necessary tension?

**Solution:** See figure for 9.8

$$\mu_k = 0.3$$

$$\sum F_x : T \cos \alpha - f - 392.4 \, N \sin 10^\circ = 0$$

$$\sum F_y : T \sin \alpha + N - 392.4 \, N \cos 10^\circ = 0$$

$$f = 0.3 \, N$$

$$\Rightarrow T = \frac{184.1 \, N}{\cos \alpha + 0.3 \sin \alpha}$$

To find the angle for minimum $T$

$$\frac{dT}{d\alpha} = \frac{184.1 \, N (\sin \alpha - 0.3 \cos \alpha)}{(\cos \alpha - 0.3 \sin \alpha)^2} = 0 \Rightarrow \tan \alpha = 0.3$$

$$\Rightarrow \alpha = 16.7^\circ$$

$$T = 176.3 \, N$$

Problem 9.10  Box A weighs 100 N, and box B weighs 30 N. The coefficients of friction between box A and the ramp are $\mu_k = 0.30$ and $\mu_k = 0.28$. What is the magnitude of the friction force exerted on box A by the ramp?

**Solution:** The sum of the forces parallel to the inclined surface is

$$\sum F = A \sin \alpha + B + f = 0,$$

from which $f = A \sin \alpha - B = 100 \sin 30^\circ - 30 = 20 \, N$
Problem 9.11  In Problem 9.10, box A weighs 100 N, and the coefficients of friction between box A and the ramp are $\mu_s = 0.30$ and $\mu_k = 0.28$. For what range of the weights of the box B will the system remain stationary?

Solution:  The upper and lower limits on the range are determined by the weight required to move the box up the ramp, and the weight that will allow the box to slip down the ramp. Assume impending slip. The friction force opposes the impending motion. For impending motion up the ramp the sum of forces parallel to the ramp are

$$\sum F = A \sin \alpha - B_{\text{MAX}} + \mu_s A \cos \alpha = 0,$$

from which

$$B_{\text{MAX}} = A(\sin \alpha + \mu_s \cos \alpha) = 100(\sin 30^\circ + 0.3 \cos 30^\circ) = 75.98 \text{ N}$$

For impending motion down the ramp:

$$\sum F = A \sin \alpha - B_{\text{MIN}} - \mu_s A \cos \alpha = 0,$$

from which

$$B = A(\sin \alpha - \mu_s \cos \alpha) = 100(\sin 30^\circ - 0.3 \cos 30^\circ) = 24.02 \text{ N}$$

Problem 9.12  The mass of the box on the left is 30 kg, and the mass of the box on the right is 40 kg. The coefficient of static friction between each box and the inclined surface is $\mu_s = 0.2$. Determine the minimum angle $\alpha$ for with the boxes will remain stationary.

Solution:  If the boxes slip when $\alpha$ is decreased, they will slip toward the right. Assume that slip toward the right impends, the free body diagrams are as shown.

The equilibrium equations are

$$\sum F_x = T - 0.2 N_A - (30)(9.81) \sin \alpha = 0,\quad (1)$$

$$\sum F_y = N_A - (30)(9.81) \cos \alpha = 0,\quad (2)$$

$$\sum F_x = -T - 0.2 N_B + (40)(9.81) \sin 30^\circ = 0,\quad (3)$$

$$\sum F_y = N_B - (40)(9.81) \cos 30^\circ = 0,\quad (4)$$

Summing Equations (1) and (3), we obtain $-0.2 N_A - 0.2 N_B - (30)(9.81) \sin \alpha + (40)(9.81) \sin 30^\circ = 0$. Solving Equation (2) for $N_A$ and Equation (4) for $N_B$ and substituting the results into Equation (5) gives $15 \sin \alpha + 3 \cos \alpha = 10 - 4 \cos 30^\circ$. Using the identity $\cos \alpha = \sqrt{1 - \sin^2 \alpha}$ and solving Equation (6) for $\sin \alpha$, we obtain $\sin \alpha = 0.242$, so $\alpha = 14.0^\circ$. 

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Problem 9.13 The coefficient of kinetic friction between the 100-kg box and the inclined surface is 0.35. Determine the tension \( T \) necessary to pull the box up the surface at a constant rate.

Solution:

\[ \sum F_x : 3T - (981 \text{ N}) \sin 60^\circ - f = 0 \]

\[ \sum F_y : N - (981 \text{ N}) \cos 60^\circ = 0 \]

\( f = 0.35 \text{ N} \)

Solving: \( T = 340 \text{ N} \)
Problem 9.14  The box is stationary on the inclined surface. The coefficient of static friction between the box and the surface is $\mu_s$.

(a) If the mass of the box is 10 kg, $\alpha = 20^\circ$, $\beta = 30^\circ$, and $\mu_s = 0.24$, what force $T$ is necessary to start the box sliding up the surface?

(b) Show that the force $T$ necessary to start the box sliding up the surface is a minimum when $\tan \beta = \mu_s$.

Solution:

Substituting the known values and solving, we get

$T = 56.5$ N,  
$N = 64.0$ N,  
$f = 15.3$ N.

Solving the 2nd equilibrium eqn for $N$ and substituting for $f$ ($f = \mu_s N$) in the first eqn, we get

$-T \cos \beta + \mu_s mg \cos \alpha = 0$

Differentiating with respect to $\beta$, we get

$\frac{dT}{d\beta} = \frac{T \sin \beta - \mu_s \cos \beta}{\cos \beta + \mu_s \sin \beta}$

Setting $\frac{dT}{d\beta} = 0$, we get

$\tan \beta = \mu_s$
**Problem 9.15** To explain observations of ship launchings at the port of Rochefort in 1779, Coulomb analyzed the system shown in Problem 9.14 to determine the minimum force $T$ necessary to hold the box stationary on the inclined surface. Show that the result is

$$T = \frac{(\sin \alpha - \mu_s \cos \alpha)mg}{\cos \beta - \mu_s \sin \beta}.$$

**Solution:**

The free-body diagram shows the forces acting on the box: the gravitational force $mg$, the normal force $N$, the friction force $f$, and the tension $T$. The forces can be resolved into components parallel and perpendicular to the inclined surface.

- The sum of forces parallel to the inclined surface gives:
  $$\sum F_x: \quad T \cos \beta + mg \sin \alpha - \mu_s N = 0$$
- The sum of forces perpendicular to the inclined surface gives:
  $$\sum F_y: \quad N + T \sin \beta - mg \cos \alpha = 0$$

Since $\alpha$ is fixed, $\beta$ is variable. Solve the second eqn for $N$ and substitute into the first. We get

$$0 = T(\mu_s \sin \beta - \cos \beta) = mg(\sin \alpha - \mu_s \cos \alpha)$$

or

$$T = \frac{mg(\sin \alpha - \mu_s \cos \alpha)}{\cos \beta - \mu_s \sin \beta}$$

To get the conditions for the minimum, set $\frac{dT}{d\beta} = 0$

$$\frac{dT}{d\beta} = \frac{T(\sin \beta + \mu_s \cos \beta)}{(\cos \beta - \mu_s \sin \beta)} = 0$$

For the min.,

$$\tan \beta = -\mu_s$$

Note $\beta$ is negative!
**Problem 9.16** Two sheets of plywood $A$ and $B$ lie on the bed of a truck. They have the same weight $W$, and the coefficient of static friction between the two sheets of wood and between sheet $B$ and the truck bed is $\mu_s$.

(a) If you apply a horizontal force to sheet $A$ and apply no force to sheet $B$, can you slide sheet $A$ off the truck without causing sheet $B$ to move? What force is necessary to cause sheet $A$ to start moving?

(b) If you prevent sheet $A$ from moving by applying a horizontal force on it, what horizontal force on sheet $B$ is necessary to start it moving?

**Solution:**

(a) The friction force exerted by sheet $A$ on $B$ at impending motion is $f_{AB} = \mu_s W$. The friction force exerted by sheet $B$ on the bed of the truck is $f_{BT} = \mu_s (2W)$, since the normal force is due to the weight of both sheets. Since $f_{BT} > f_{AB}$, the top sheet will begin moving before the bottom sheet. Yes

The force required to start sheet $A$ to move is

$$F = f_{AB} = \mu_s W.$$

(b) The force on $B$ is the friction between $A$ and $B$ and the friction between $B$ and the truck bed. Thus the force required to start $B$ in motion is

$$F_B = f_{AB} + f_{BT} = 3\mu_s W.$$

**Problem 9.17** The weights of the two boxes are $W_1 = 100 \text{ N}$ and $W_2 = 50 \text{ N}$. The coefficients of kinetic friction between the left box and the inclined surface are $\mu_s = 0.12$ and $\mu_k = 0.10$. Determine the tension the man must exert on the rope to pull the boxes upward at a constant rate.

**Solution:**

$$\sum F_x: T - 100 \text{ N} \sin 30^\circ - f - 50 \text{ lb} = 0$$

$$\sum F_y: N - 100 \text{ N} \cos 30^\circ = 0$$

$$f = 0.10 \text{ N}$$

Solving: $T = 109 \text{ N}$
Problem 9.18  In Problem 9.17, for what range of tensions exerted on the rope by the man will the boxes remain stationary?

Solution:  See the figure in 9.17.
First solve for the largest force $T_{max}$
$$\sum F_1 = T_{max} - 100 \text{ N} \sin 30^\circ - f - 50 \text{ N} = 0$$
$$\sum F_2 = N - 100 \text{ N} \cos 30^\circ = 0 \quad \Rightarrow T_{max} = 110.4 \text{ N}$$
$$f = 0.12 \text{ N}$$
Next solve for the smallest force $T_{min}$. We need to turn the friction force in the opposite direction.
$$\sum F_1 = T_{min} - 100 \text{ N} \sin 30^\circ + f - 50 \text{ N} = 0$$
$$\sum F_2 = N - 100 \text{ N} \cos 30^\circ = 0 \quad \Rightarrow T_{min} = 89.6 \text{ N}$$
$$f = 0.12 \text{ N}$$
Thus for the boxes to remain stationary we must have
$$89.6 \text{ N} < T < 110.4 \text{ N}$$

Problem 9.19  Each box weighs 10 N. The coefficient of static friction between box $A$ and box $B$ is 0.24, and the coefficient of static friction between box $B$ and the inclined surface is 0.3. What is the largest angle $\alpha$ for which box $B$ will not slip?

Strategy:  Draw individual free-body diagrams of the two boxes and write their equilibrium equations assuming that slip of box $B$ is impending.

Solution:  We have 6 unknowns, 4 equilibrium equations and 2 friction equations
$$\sum F_{A1} = T - 10 \text{ N} \sin \alpha - f_2 = 0$$
$$\sum F_{A2} = N_2 - 10 \text{ N} \cos \alpha = 0$$
$$\sum F_{B1} = f_2 + f_1 - 10 \text{ N} \sin \alpha = 0$$
$$\sum F_{B2} = N_1 - N_2 - 10 \text{ N} \cos \alpha = 0$$
$$f_1 = 0.3N_1, \quad f_2 = 0.24N_2$$
Solving we find $\alpha = 40.0^\circ$
Problem 9.20 The masses of the boxes are $m_A = 15 \text{ kg}$ and $m_B = 60 \text{ kg}$. The coefficient of static friction between boxes $A$ and $B$ and between box $B$ and the inclined surface is 0.12. What is the largest force $F$ for which the boxes will not slip?

**Solution:** We have 6 unknowns, 4 equilibrium equations and 2 friction equations.

\[
\begin{align*}
\sum F_A : T - F - (147.15 \text{ N}) \sin 20^\circ + f_2 &= 0 \\
\sum F_A : N_2 - (147.15 \text{ N}) \cos 20^\circ &= 0 \\
\sum F_B : T - (588.6 \text{ N}) \sin 20^\circ - f_1 - f_2 &= 0 \\
\sum F_B : N_1 - N_2 - (588.6 \text{ N}) \cos 20^\circ &= 0 \\
f_1 &= 0.12N_1, \quad f_2 = 0.12N_2 \\
\end{align*}
\]

Solving we find $F = 267 \text{ N}$

Problem 9.21 In Problem 9.20, what is the smallest force $F$ for which the boxes will not slip?

**Solution:** See the solution for 9.20 — change the directions of all of the friction forces.

\[
\begin{align*}
\sum F_A : T - F - (147.15 \text{ N}) \sin 20^\circ - f_2 &= 0 \\
\sum F_A : N_2 - (147.15 \text{ N}) \cos 20^\circ &= 0 \\
\sum F_B : T - (588.6 \text{ N}) \sin 20^\circ + f_1 + f_2 &= 0 \quad \Rightarrow \quad F = 34.8 \text{ N} \\
\sum F_B : N_1 - N_2 - (588.6 \text{ N}) \cos 20^\circ &= 0 \\
f_1 = 0.12N_1, \quad f_2 = 0.12N_2 \\
\end{align*}
\]
Problem 9.22  In Example 9.2, what clockwise couple \( M \) would need to be applied to the disk to cause it to rotate at a constant rate in the clockwise direction?

Solution: Assume that the disk is rotating in the clockwise direction. From the free-body diagram of the disk,
\[
\Sigma M_D = -M + (R \sin \theta_1) r = 0.
\]
From the free-body diagram of the brake,
\[
\Sigma M_A : -F (\frac{1}{2} h) + (R \cos \theta_1) b + (R \sin \theta_1) h = 0.
\]
Solving these two equations yields
\[
M = \frac{1}{2} h b F \mu_k \frac{h + b \mu_k}{h}\.
\]

Problem 9.23  The homogeneous horizontal bar \( AB \) weighs 200 N. The homogeneous disk weighs 300 N. The coefficient of kinetic friction between the disk and the sloping surface is \( \mu_k = 0.24 \). What is the magnitude of the couple that would need to be applied to the disk to cause it to rotate at a constant rate in the clockwise direction?

Solution: From the free-body diagram of the bar,
\[
\Sigma M_B : (200 \text{ N}) (2.5 \text{ m}) + A_y (5 \text{ m}) = 0
\]
\[
\Rightarrow A_y = -100 \text{ N}.
\]
From the free-body diagram of the disk,
\[
\Sigma F_x : A_x + N \sin 20^\circ + \mu_k N \cos 20^\circ = 0,
\]
\[
\Sigma F_y : N \cos 20^\circ - \mu_k N \sin 20^\circ - 300 \text{ N} = 0,
\]
\[
\Sigma M_A : -M + \mu_k N (1 \text{ m}) = 0.
\]
Solving yields \( A_x = -265 \text{ N} \), \( N = 466 \text{ N} \), \( M = 112 \text{ N} \cdot \text{m} \).
\[
M = 112 \text{ N} \cdot \text{m}.
\]
Problem 9.24  The homogeneous horizontal bar $AB$ weighs 200 N. The homogeneous disk weighs 300 N. The coefficient of kinetic friction between the disk and the sloping surface is $\mu_k = 0.24$. What is the magnitude of the couple that would need to be applied to the disk to cause it to rotate at a constant rate in the counterclockwise direction?

Solution: From the free-body diagram of the bar,

\[ \Sigma M_B : (200 \text{ N})(2.5 \text{ m}) + A_y(5 \text{ m}) = 0 \]

\[ \Rightarrow A_y = -100 \text{ N} \]

From the free-body diagram of the disk,

\[ \Sigma F_x : A_x + N \sin 20^\circ - \mu_k N \cos 20^\circ = 0, \]

\[ \Sigma F_y : N \cos 20^\circ + \mu_k N \sin 20^\circ - 300 \text{ N} = 0, \]

\[ \Sigma M_A : M - \mu_k N(1 \text{ m}) = 0. \]

Solving yields $A_x = -45.6 \text{ N}, N = 391 \text{ N}, M = 94 \text{ N-m}.$

Problem 9.25  The mass of the bar is 4 kg. The coefficient of static friction between the bar and the floor is 0.3. Neglect friction between the bar and the wall.

(a) If $\alpha = 20^\circ$, what is the magnitude of the friction force exerted on the bar by the floor?

(b) What is the maximum angle $\alpha$ for which the bar will not slip?

Solution:

(a) $\alpha = 20^\circ$

\[ f_{\text{max}} = 0.3 N_B \]

\[ \Sigma F_x : N_A - f_B = 0 \]

\[ \Sigma F_y : N_B - 39.24 \text{ N} = 0 \]

\[ \Sigma M_B : (39.24 \text{ N})(0.5 \text{ m}) \sin \alpha - N_A(1.0 \text{ m}) \cos \alpha = 0 \]

Solving: $f_B = 7.14 \text{ N}, f_{\text{max}} = 11.77 \text{ N}$

Since $f_B < f_{\text{max}}, f_B = 7.14 \text{ N}$

(b) $f_B = 0.3 N_B$

\[ \Sigma F_x : N_A - f_B = 0 \]

\[ \Sigma F_y : N_B - 39.24 \text{ N} = 0 \]

\[ \Sigma M_B : (39.24 \text{ N})(0.5 \text{ m}) \sin \alpha - N_A(1.0 \text{ m}) \cos \alpha = 0 \]

\[ \Rightarrow \alpha = 31.0^\circ \]
Problem 9.26  In Problem 9.25, suppose that the coefficient of static friction between the bar and the floor and between the 4-kg bar and the wall is 0.3. What is the maximum angle \( \alpha \) for which the bar will not slip?

Solution:

\[ f_B = 0.3N_B, \quad f_A = 0.3N_A \]

\[ \sum F_x : N_A - f_B = 0 \]

\[ \sum F_y : N_B + f_A - 39.24 \text{ N} = 0 \]

\[ \sum M_B : (39.24 \text{ N})(0.5 \text{ m}) \sin \alpha - N_A (1.0 \text{ m}) \cos \alpha \]

\[ - f_A (1.0 \text{ m}) \sin \alpha = 0 \]

Solving \( \alpha = 33.4^\circ \)
Problem 9.27  The ladder and the person weigh 150 N and 900 N, respectively. The center of mass of the 3.6 m ladder is at its midpoint. The angle $\alpha = 30^\circ$. Assume that the wall exerts a negligible friction force on the ladder.

(a) If $x = 1.2$ m, what is the magnitude of the friction force exerted on the ladder by the floor?

(b) What minimum coefficient of static friction between the ladder and the floor is necessary for the person to be able to climb to the top of the ladder without slipping?

Solution:

(a) Assume no slipping occurs

\[\alpha = 30^\circ, \ x = 1.2 \ m\]

\[\sum F_x : f_B - N_A = 0\]

\[\sum F_y : N_B - 1050 \ N = 0\]

\[\sum M_B : N_A (3.6 \ m \ \cos \alpha) - 150 (1.8 \ m \ \sin \alpha) - 900 \ N \ x = 0\]

Solving \[f_B = 389.7 \ N, \ N_B = 1050 \ N\]

(b) At the top of the ladder

\[\alpha = 30^\circ, \ x = 1.8 \ m\]

\[\sum F_x : f_B - N_A = 0\]

\[\sum F_y : N_B - 1050 \ N = 0\]

\[\sum M_B : N_A (3.6 \ m \ \cos \alpha) - 150 (1.8 \ m \ \sin \alpha) - 900 \ N \ x = 0\]

\[f_B = \mu_s N_B\]

\[\Rightarrow \mu_s = 0.536\]
Problem 9.28  In Problem 9.27, the ladder and the person weigh 150 N and 900 N, respectively. The center of mass of the 3.6 m ladder is at its midpoint. The coefficient of static friction between the ladder and the floor is $\mu_s = 0.5$. What is the largest value of the angle $\alpha$ for which the person could climb to the top of the ladder without it slipping?

Solution:  See the figure for Problem 9.27.

\[
x = (3.6 \text{ m}) \sin \alpha
\]

\[
\sum F_x : f_B - N_A = 0
\]

\[
\sum F_y : N_B - 1050 \text{ N} = 0
\]

\[
\sum M_B : N_A (3.6 \cos \alpha) - 150 \text{ N} (1.8 \sin \alpha) - 900 \text{ N} x = 0
\]

\[f_B = 0.5N_B\]

\[\Rightarrow \alpha = 28.3^\circ\]

Problem 9.29  In Problem 9.27, the ladder and the person weigh 150 N and 900 N, respectively. The center of mass of the 3.6 m ladder is at its midpoint. The coefficient of static friction between the ladder and the floor is 0.5 and the coefficient of friction between the ladder and the wall is 0.3. What is the largest value of the angle $\alpha$ for which the person could climb to the top of the ladder without it slipping? Compare your answer to the answer to Problem 9.28.

Solution:

\[
x = (3.6 \text{ m}) \sin \alpha
\]

\[
\sum F_x : f_B - N_A = 0
\]

\[
\sum F_y : N_B - 1050 \text{ N} + f_A = 0
\]

\[
\sum M_B : N_A (3.6 \cos \alpha) - 150 \text{ N} (1.8 \sin \alpha)
\]

\[-900 \text{ N} x + f_Ax = 0
\]

\[f_B = 0.5N_B, \ f_A = 0.3N_A\]

Solving \[\alpha = 28.6^\circ\]

Friction at the wall didn’t help much.
Problem 9.30  The disk weighs 50 N and the bar weighs 25 N. The coefficients of friction between the disk and the inclined surface are $\mu_s = 0.6$ and $\mu_k = 0.5$.

(a) What is the largest couple $M$ that can be applied to the stationary disk without causing it to start rotating?
(b) What couple $M$ is necessary to rotate the disk at a constant rate?

Solution:  The problem has 7 unknowns, 6 equilibrium equations and one friction equation.

(a)  No slip
\[ \sum M_A : (25 \text{ N} \times 10 \cos 30^\circ) - R_2 (20 \text{ cm}) = 0 \]
\[ \sum F_x : N - R_2 - 50 \text{ N} \cos 30^\circ = 0 \]
\[ \sum M_B : M - f (5 \text{ cm}) = 0 \]
\[ f = 0.6 \text{ N} \]
\[ \Rightarrow M = 162.4 \text{ N} \cdot \text{cm} \]

(b)  Steady rotation. Replace the last equation with
\[ f = 0.5 \text{ N} \Rightarrow M = 135.3 \text{ N} \cdot \text{cm} \]

Problem 9.31  The radius of the 40-kg homogeneous cylinder is $R = 0.15 \text{ m}$. The slanted wall is smooth and the angle $\alpha = 30^\circ$. The coefficient of static friction between the cylinder and the floor is $\mu_s = 0.2$. What is the largest couple $M$ that can be applied to the cylinder without causing it to slip?

Solution:  Assume that slip of the wheel is impending. The equilibrium equations are
\[ \Sigma F_x : P \sin \alpha - \mu_s N = 0, \]
\[ \Sigma F_y : N + P \cos \alpha - W = 0, \]
\[ \Sigma M_{center} : M - \mu_s NR = 0 \]
where $W = (40 \text{ kg}) (9.81 \text{ m/s}^2)$. Putting in the values for $W, \alpha, R, \mu_s$ and solving yields
\[ P = 117 \text{ N}, N = 291 \text{ N}, M = 8.74 \text{ N} \]
\[ M = 8.74 \text{ N} \cdot \text{m} \]
**Problem 9.32** The homogeneous cylinder has weight \( W \). The coefficient of static friction between the cylinder and both surfaces is \( \mu_s \). What is the largest couple \( M \) that can be applied to the cylinder without causing it to slip? (Assume that the cylinder slips before rolling up the inclined surface.)

**Solution:** Assume that slip of the wheel is impending. The equilibrium equations are

\[
\begin{align*}
\sum F_x & : P \sin \alpha - \mu_s P \cos \alpha - \mu_s N = 0, \\
\sum F_y & : N + P \cos \alpha + P \sin \alpha - W = 0, \\
\sum M_{	ext{center}} & : M - \mu_s NR - \mu_s PR = 0
\end{align*}
\]

Solving these equations for \( P, N, \) and \( M \), we obtain

\[
M = \frac{\mu_s RW (\sin \alpha + \mu_s (1 - \cos \alpha))}{(1 + \mu_s^2) \sin \alpha}
\]

**Problem 9.33** The homogeneous cylinder has weight \( W \). The coefficient of static friction between the cylinder and both surfaces is \( \mu_s \). What is the minimum value of \( \mu_s \) for which the couple \( M \) will cause the cylinder to roll up the inclined surface without slipping?

**Solution:** Assume that slip of the cylinder is impending and the cylinder is on the verge of rolling up the inclined surface (the normal force exerted by the floor is zero). The equilibrium equations are

\[
\begin{align*}
\sum F_x & : P \sin \alpha - \mu_s P \cos \alpha = 0, \\
\sum F_y & : P \cos \alpha + \mu_s P \sin \alpha - W = 0, \\
\sum M_{	ext{center}} & : M - \mu_s PR = 0
\end{align*}
\]

From the first equation we see that

\[\mu_s = \tan \alpha.\]
Problem 9.34  The coefficient of static friction between the blades of the shears and the object they are gripping is 0.36. What is the largest value of the angle \( \alpha \) for which the object will not slip out? Neglect the object’s weight.

Strategy: Draw the free-body diagram of the object and assume that slip is impending.

Solution:
\[
\sum F_x = 2f \cos(\alpha/2) - 2N \sin(\alpha/2) = 0
\]
\[
f = 0.36N
\]
Solving
\[
\tan(\alpha/2) = 0.36 \Rightarrow \alpha = 39.6^\circ
\]

Problem 9.35  The stationary disk of 300-mm radius is attached to a pin support at \( D \). The disk is held in place by the brake \( ABC \) in contact with the disk at \( C \). The hydraulic actuator \( BE \) exerts a horizontal 400-N force on the brake at \( B \). The coefficients of friction between the disk and the brake are \( \mu_s = 0.6 \) and \( \mu_k = 0.5 \). What couple must be applied to the stationary disk to cause it to slip in the counterclockwise direction?

Solution: Assume impending slip. For counterclockwise motion the friction force \( f = \mu_k F_N \) opposes the impending slip, so that it acts on the brake in a downward direction, producing a negative moment (clockwise) about \( A \). The sum of the moments about \( A \):
\[
\sum M_A = -0.2(400) + (0.4 - 0.2\mu_k)F_N,
\]
from which \( F_N = 285.7 \) N. The sum of the moments about the center of the disk:
\[
\sum M_D = M - 0.3(\mu_k)F_N = 0,
\]
from which \( M = 51.43 \) N m.
Problem 9.36  The figure shows a preliminary conceptual idea for a device to exert a braking force on a rope when the rope is pulled downward by the force $T$. The coefficient of kinetic friction between the rope and the two bars is $\mu_k = 0.28$. Determine the force $T$ necessary to pull the rope downward at a constant rate if $F = 10$ N and (a) $\alpha = 30^\circ$; (b) $\alpha = 20^\circ$.

Solution: From the rope we see that $T = 2f$

From one of the boards we have

$\sum M_A : -(10 \text{ N})(3 \text{ cm} \cos \alpha) + N(6 \text{ cm} \sin \alpha) - f(6 \text{ cm} \cos \alpha) = 0$

On the verge of slipping we have

$f = 0.28 \text{ N}$

(a) Solving if $\alpha = 30^\circ$ $T = 9.42 \text{ N}$

(b) Solving if $\alpha = 20^\circ$ $T = 33.3 \text{ N}$
Problem 9.37  The mass of block $B$ is 8 kg. The coefficient of static friction between the surfaces of the clamp and the block is $\mu_s = 0.2$. When the clamp is aligned as shown, what minimum force must the spring exert to prevent the block from slipping out?

Solution: The free-body diagram of the block when slip is impending is shown. From the equilibrium equation

$$\mu_s F_T + \mu_s (F_T + W \cos \alpha) - W \cos \alpha = 0,$$

we obtain

$$F_T = \frac{\mu_s (1 - \mu_s) \cos \alpha}{2 \mu_s}$$

$$= \frac{(8)(9.1)(1 - 0.2) \cos 45}{2(0.2)}$$

$$= 111 \text{ N}.$$

The free-body diagram of the upper arm of the clamp is shown. Summing moments about the upper end,

$$0.16 F_s + 0.1 \mu_s F_T - 0.36 F_T = 0,$$

the force exerted by the spring is

$$F_s = \frac{0.36 F_T - 0.1 \mu_s F_T}{0.16}$$

$$= \frac{[0.36 - 0.1(0.2)]111}{0.16}$$

$$= 236 \text{ N}.$$

Problem 9.38  By altering its dimensions, redesign the clamp in Problem 9.37 so that the minimum force the spring must exert to prevent the block from slipping out is 180 N. Draw a sketch of your new design.

Solution: This problem does not have a unique solution.
Problem 9.39  The horizontal bar is attached to a collar that slides on the smooth vertical bar. The collar at P slides on the smooth horizontal bar. The total mass of the horizontal bar and the two collars is 12 kg. The system is held in place by the pin in the circular slot. The pin contacts only the lower surface of the slot, and the coefficient of static friction between the pin and the slot is 0.8. If the system is in equilibrium and \( y = 260 \) mm, what is the magnitude of the friction force exerted on the pin by the slot?

Solution:  The free-body diagram of the horizontal bar and right collar is as shown, where \( m_1 \) is the mass of the horizontal bar and right collar, \( N_1 \) is the normal force exerted by the vertical bar, and \( N_2 \) is the force exerted by the left collar. From the equilibrium equations:

\[
\sum F_x = -N_1 = 0, \\
\sum F_y = N_2 - m_1 g = 0,
\]

we see that \( N_2 = m_1 g \). The free body diagram of the left collar is as shown, where \( m_2 \) is the mass of the left collar and \( N \), \( f \) are the normal and friction forces exerted by the curved slot.

\( y = 260 \) mm = \((300 \) mm\) \( \sin \theta \),
so the angle \( \theta = 60.1^\circ \).

From the equilibrium equations,

\[
\sum F_x = -f + m_2 g \cos \theta + N_2 \cos \theta = 0, \\
\sum F_y = N - m_2 g \sin \theta - N_2 \sin \theta = 0,
\]

we obtain

\[
f = (m_2 g + N_2) \cos \theta = (m_2 + m_1) \cos \theta = (12)(9.81) \cos 60.1^\circ \\
= 58.7 \text{ N}.
\]

Problem 9.40  In Problem 9.39, what is the minimum height \( y \) at which the system can be in equilibrium?

Solution:  From the solution of Problem 9.39, the friction and normal forces exerted on the pin by the circular slot are

\[
f = (m_2 g + N_2) \cos \theta, \\
N = (m_2 g + N_2) \sin \theta,
\]

so \( \frac{f}{N} = \cot \theta \). When slip impends,

\[
f = \mu N = 0.8 N,
\]

so 0.8 = \( \cot \theta \) and \( \theta = 51.3^\circ \). The height \( y = 300 \sin \theta = 234 \) mm.
Problem 9.41  The rectangular 100-N plate is supported by the pins A and B. If friction can be neglected at A and the coefficient of static friction between the pin at B and the slot is \( \mu_s = 0.4 \), what is the largest angle \( \alpha \) for which the plate will not slip?

Solution:  Choose a coordinate system with the \( x \) axis parallel to the rail. The sum of the moments about \( A \) is

\[
\sum M_A = -0.48W \cos \alpha - 0.54W \sin \alpha + 0.96R = 0,
\]

from which

\[
R = \frac{W}{0.96} (0.48 \cos \alpha + 0.54 \sin \alpha).
\]

The component of weight causing the plate to slide is \( F = W \sin \alpha \). This must be balanced by the friction force: \( 0 = -W \sin \alpha + \mu_s R \), from which

\[
W \sin \alpha = \frac{W}{0.96} (0.48 \cos \alpha + 0.54 \sin \alpha).
\]

Reduce algebraically to obtain

\[
\alpha = \tan^{-1} \left( \frac{\mu_s}{2 - 1.125 \mu_s} \right) = 14.47^\circ.
\]

Problem 9.42  If you can neglect friction at B in Problem 9.41 and the coefficient of static friction between the pin at A and the slot is \( \mu_s = 0.4 \), what is the largest angle \( \alpha \) for which the 100-N plate will not slide?

Solution:  The normal force acts normally to the slots, and the friction force acts parallel to the slot. Choose a coordinate system with the \( x \) axis parallel to the slots. The normal component of the reaction at \( A \) is found from the sum of the moments about \( B \):

\[
\sum M_B = -0.54W \sin \alpha + 0.48W \cos \alpha - 0.96A_N = 0,
\]

from which

\[
A_N = \frac{W}{0.96} (-0.54 \sin \alpha + 0.48 \cos \alpha).
\]

The force tending to make the plate slide is \( F = -W \sin \alpha \). This is balanced by the friction force at \( A \):

\[
0 = -W \sin \alpha + \mu_s A_N,
\]

from which

\[
W \sin \alpha = \frac{W}{0.96} (-0.54 \sin \alpha + 0.48 \cos \alpha).
\]

Reduce algebraically to obtain

\[
\alpha = \tan^{-1} \left( \frac{\mu_s}{2 + 1.125 \mu_s} \right) = 9.27^\circ.
\]

Check: The normal reactions at \( A \) and \( B \) are unequal: as the slots are inclined from the horizontal, the parallel component of the gravity force reduces the normal force at \( A \), and increases the normal force at \( B \). check.

Check: The sum of the reactions at \( A \) and \( B \) are \( A_N + B_N = W \cos \alpha \). check. The magnitude \( \sqrt{(A_N + B_N)^2 + (\mu_s A_N)^2} = W \), hence the system is in equilibrium at impending slip. check.
Problem 9.43 The airplane's weight is $W = 12 \text{kN}$. Its brakes keep the rear wheels locked, and the coefficient of static friction between the wheels and the runway is $\mu_s = 0.6$. The front (nose) wheel can turn freely and so exerts only a normal force on the runway. Determine the largest horizontal thrust force $T$ the plane's propeller can generate without causing the rear wheels to slip.

Solution: The free body diagram when slip of the rear wheels impends is shown. From the equilibrium equations

\[ \sum f_x = -T + \mu_s B = 0, \]
\[ \sum f_y = A + B - W = 0, \]
\[ \sum M_{pt} = 1.2T - 1.5W + 2.1B = 0, \]

we obtain

\[ A = 5617 \text{ N}, \]
\[ B = 6383 \text{ N}, \]

and $T = 3830 \text{ N}$.

Problem 9.44 The refrigerator weighs 900 N. It is supported at $A$ and $B$. The coefficient of static friction between the supports and the floor is $\mu_s = 0.2$. If you assume that the refrigerator does not tip over before it slips, what force $F$ is necessary for impending slip?

Solution: Assume that slip is impending. From the equilibrium equations

\[ \Sigma F_x : F - \mu_s A - \mu_s B = 0, \]
\[ \Sigma F_y : A + B - W = 0. \]

Solving we find

\[ F = \mu_s (A + B) = \mu_s W = (0.2)(900 \text{ N}) = 180 \text{ N}. \]
**Problem 9.45** The refrigerator weighs 900 N. It is supported at A and B. The coefficient of static friction between the supports and the floor is $\mu_s = 0.2$. The distance $h = 1.5$ m and the dimension $b = 0.75$ m. When the force $F$ is applied to push the refrigerator across the floor, will it tip over before it slips? (See Example 9.3.)

**Solution:** See the analysis in Example 9.3. The refrigerator will tip over before it slips if

$$h > \frac{b}{2\mu_s} = \frac{30 \text{ m}}{2(0.2)} = 75 \text{ m}.$$ 

No.
Problem 9.46  To obtain a preliminary evaluation of the stability of a turning car, imagine subjecting the stationary car to an increasing lateral force $F$ at the height of its center of mass, and determine whether the car will slip (skid) laterally before it tips over. Show that this will be the case if $b/h > \frac{2}{\mu_s}$. (Notice the importance of the height of the center of mass relative to the width of the car. This reflects on recent discussions of the stability of sport utility vehicles and vans that have relatively high centers of mass.)

**Solution:**

For $b/h > \frac{2}{\mu_s}$, slip before tip

For $b/h < \frac{2}{\mu_s}$, tip before slip

For $b/h$ big $N$ low cm, relative to track width

For $b/h$ small $N$ high cm, relative to track width

Equilibrium Eqs.

$\sum F_x: \quad F - f_L - f_R = 0$

$\sum F_y: \quad N_L + N_R - mg = 0$

$\sum M_x: \quad -hF + bN_R - \frac{b}{2}mg = 0$

Assume skid and tip simultaneously.

$f_L = \mu_s N_L$

$f_R = \mu_s N_R$ (skid)

and $N_L = 0$ (tip), $\therefore f_L = 0$

$f_R = \mu_s mg$

The equilibrium eqns become

$F = f_R = \mu_s N_R = \mu_s mg$

and the moment eqn. uses

$-h(\mu_s mg) + b(mg) - \frac{b}{2}mg = 0$

or $\frac{b}{h} = \frac{2}{\mu_s}$
**Problem 9.47** The man exerts a force $P$ on the car at an angle $\alpha = 20^\circ$. The 1760-kg car has front wheel drive. The driver spins the front wheels, and the coefficient of kinetic friction is $\mu_k = 0.02$. Snow behind the rear tires exerts a horizontal resisting force $S$. Getting the car to move requires overcoming a resisting force $S = 420$ N. What force $P$ must the man exert?

**Solution:**

\[
\sum F_x : \quad S - \mu_k N_F - P \cos \alpha = 0
\]
\[
\sum F_y : \quad N_F + mg - P \sin \alpha = 0
\]
\[
\sum M_A : \quad -(1.62)mg + 2.55 N_F - (0.90)P \cos \alpha - (3.40)P \sin \alpha = 0
\]

$\alpha = 20^\circ$,

$m = 1760$ kg,

$g = 9.81$ m/s$^2$,

$S = 420$ N,

$\mu_k = 0.02$,

3 eqns in 3 unknowns ($N_R$, $N_F$, and $P$)

Solving the equations, we get $P = 213$ N

$N_R = 6.34$ kN

$N_F = 11.00$ kN
Problem 9.48 In Problem 9.47, what value of the angle \( \alpha \) minimizes the magnitude of the force \( P \) the man must exert to overcome the resisting force \( S = 420 \text{ N} \) exerted on the rear tires by the snow? What force must he exert?

Solution: From the solution to Problem 9.47, we have

\[ S - \mu_k N_F - P \cos \alpha = 0 \]  
(1)

\[ N_F + N_e - mg - P \sin \alpha = 0 \]  
(2)

\[-1.62 mg + 2.55 N_F + 0.90P \cos \alpha - 3.40P \sin \alpha = 0 \]  
(3)

where

\[ \mu_k = 0.02, \]
\[ S = 420 \text{ N}, \]
\[ m = 1760 \text{ kg}, \]

and \( g = 9.81 \text{ m/s}^2 \).

From Eqn (1),

\[ N_F = \frac{1}{\mu_k} (S - P \cos \alpha) \quad (a) \]

From Eqn (2),

\[ N_e = N_F + mg + P \sin \alpha \]

or

\[ N_e = \frac{1}{\mu_k} (S - P \cos \alpha) + mg + P \sin \alpha \quad (b) \]

Substitute (a) and (b) into (3)

We get

\[-1.62 mg + 2.55 \left( \frac{1}{\mu_k} \right) (S - P \cos \alpha) + 0.90 P \cos \alpha - 3.40 P \sin \alpha = 0 \]

Use this eqn to find \( \frac{dP}{d\alpha} \) and set it to zero.

\[ \frac{2.55}{\mu_k} \left[ \frac{dP}{d\alpha} \cos \alpha + P \sin \alpha \right] + \frac{0.90}{\mu_k} \frac{dP}{d\alpha} \cos \alpha - 0.90 \frac{dP}{d\alpha} \sin \alpha - 3.40 \frac{dP}{d\alpha} \sin \alpha - 3.40 P \cos \alpha = 0 \]

or

\[ \frac{dP}{d\alpha} \left[ 0.90 \cos \alpha - 3.40 \sin \alpha - \frac{2.55}{\mu_k} \cos \alpha \right] + P \left[ \frac{2.55}{\mu_k} \sin \alpha - 0.90 \sin \alpha - 3.40 \cos \alpha \right] = 0 \]

\[ \frac{dP}{d\alpha} = 0 = -P \left[ \frac{2.55}{\mu_k} - 0.90 \right] \sin \alpha - 3.40 \cos \alpha \]

\[ \tan \alpha = \frac{3.40}{\frac{2.55}{\mu_k} - 0.90} \]

Solving, \( \alpha = 1.54^\circ \)

Substituting this back into Eqns (1), (2), and (3), and solving, we get

\[ P = 202 \text{ N} \]
**Problem 9.49** The coefficient of static friction between the 15 kN car’s tires and the road is $\mu_s = 0.5$. Determine the steepest grade (the largest value of the angle $\alpha$) the car can drive up at constant speed if the car has (a) rear-wheel drive; (b) front-wheel drive; (c) four-wheel drive.

**Solution:** The friction force acts parallel to the incline, and the normal force is normal to the incline. Choose a coordinate system with the $x$ axis parallel to the incline. The component of the weight that acts parallel to the incline is $W \sin \alpha$, and the component acting normally to the incline is $W \cos \alpha$.

(a) **For rear wheel drive:** The moment about the point of contact of the front wheels:

$$\sum M_{FW} = 0.875 W \cos \alpha + 0.475 W \sin \alpha - 2.675 R = 0,$$

from which the normal reaction of the two rear wheels is

$$R = \frac{W}{2.675} (0.875 \cos \alpha + 0.475 \sin \alpha).$$

The force causing impending slip is $W \sin \alpha$, which is balanced by the friction force: $0 = W \sin \alpha - \mu_s R$, from which

$$\frac{W \sin \alpha}{\mu_s} = \frac{W}{2.675} (0.875 \cos \alpha + 0.475 \sin \alpha).$$

Reduce and solve:

$$\alpha = \tan^{-1} \left( \frac{0.875}{2.675 \mu_s + 0.475} \right) = 10.18^\circ$$

is the maximum angle at impending slip.

(b) **For front wheel drive:** The moments about the point of contact of the rear wheels is

$$\sum M_{RW} = -1.8 W \cos \alpha + 0.475 W \sin \alpha + 2.675 F = 0,$$

from which the normal reaction of the two front wheels is

$$F = \frac{W}{2.675} (1.8 \cos \alpha - 0.475 \sin \alpha).$$

The friction force balances the component of gravity parallel to the incline: $0 = -W \sin \alpha + \mu_s F$, from which

$$\frac{W \sin \alpha}{\mu_s} = \frac{W}{2.675} (1.8 \cos \alpha - 0.475 \sin \alpha).$$

Reduce and solve:

$$\alpha = \tan^{-1} \left( \frac{1.8}{2.675 \mu_s + 0.475} \right) = 17.17^\circ$$

(c) **For four wheel drive:** Use the reactions of the front and rear wheels obtained in Parts (a) and (b). The sum of the forces parallel to the incline is

$$\sum F_x = -W \sin \alpha + \mu_s R + \mu_s F = 0,$$

from which

$$W \sin \alpha = \frac{W}{2.675} (0.875 \cos \alpha + 0.475 \sin \alpha + 1.8 \cos \alpha - 0.475 \sin \alpha).$$

Reduce and solve: $\alpha = \tan^{-1}(\mu_s) = 26.57^\circ$

Check: This result is the same as if the Mercedes with four wheel drive were a box on an incline, as it should be.
Problem 9.50  The stationary cabinet has weight $W$. Determine the force $F$ that must be exerted to cause it to move if (a) the coefficient of static friction at $A$ and $B$ is $\mu_sS$; (b) if the coefficient of static friction at $A$ is $\mu_{sA}$ and the coefficient of static friction at $B$ is $\mu_{sB}$.

Solution:  (a) The sum of the moments about $B$ is

$$\sum M_B = -bF + \left(\frac{b}{2}\right) W - W = 0,$$

from which

$$A = \frac{W}{2} - \left(\frac{b}{2}\right) F.$$

The sum of forces:

$$\sum F_y = -W + A + B = 0,$$

from which

$$B = W - A = \frac{W}{2} + \left(\frac{b}{2}\right) F.$$

$$\sum F_x = F - \mu_sA - \mu_sB = 0,$$

from which

$$F = \mu_s \left(\frac{W}{2} + \left(\frac{b}{2}\right) F + \frac{W}{2} - \left(\frac{b}{2}\right) F\right) = \mu_sW,$$

(b) Use the normal reactions found in Part (a). From the sum of forces parallel to the floor,

$$F = \mu_{sA} A + \mu_{sB} B = \mu_{sA} \left(\frac{W}{2} - \left(\frac{b}{2}\right) F\right) + \mu_{sB} \left(\frac{W}{2} + \left(\frac{b}{2}\right) F\right).$$

Reduce and solve:

$$F = \frac{W}{\frac{1}{2} \left(\mu_{sA} + \mu_{sB}\right)} \left(1 + \frac{b}{W} \left(\mu_{sA} - \mu_{sB}\right)\right).$$
Problem 9.51 The table weighs 250 N and the coefficient of static friction between its legs and the inclined surface is 0.7.

(a) If you apply a force at A parallel to the inclined surface to push the table up the inclined surface, will the table tip over before it slips? If not, what force is required to start the table moving up the surface?

(b) If you apply a force at B parallel to the inclined surface to push the table down the inclined surface, will the table tip over before it slips? If not, what force is required to start the table moving down the surface?

Solution:

(a) Assume the table does not tip

\[ \sum F_x = F - f_1 - f_2 - 250 \text{ N} \sin 20^\circ = 0 \]

\[ \sum F_y = N_1 + N_2 - 250 \text{ N} \cos 20^\circ = 0 \]

\[ \sum M_2 = -F(0.8 \text{ m}) - N_1(1.15 \text{ m}) + (250 \text{ N} \cos 20^\circ)(0.575 \text{ m}) + (250 \text{ N} \sin 20^\circ)(0.7 \text{ m}) = 0 \]

\[ f_1 = 0.7N_1, \quad f_2 = 0.7N_2 \]

Solving we find \( F = 250 \text{ N}, \quad N_1 = -4.37 \text{ N}, \quad N_2 = 239.3 \text{ N} \)

Since \( N_1 < 0 \) we conclude that the table tips before it slips.

(b) Assume the table does not tip

\[ \sum F_x = -F + f_1 + f_2 - 250 \text{ N} \sin 20^\circ = 0 \]

\[ \sum F_y = N_1 + N_2 - 250 \text{ N} \cos 20^\circ = 0 \]

\[ \sum M_2 = F(0.8 \text{ m}) - N_1(1.15 \text{ m}) + (250 \text{ N} \cos 20^\circ)(0.575 \text{ m}) + (250 \text{ N} \sin 20^\circ)(0.7 \text{ m}) = 0 \]

\[ f_1 = 0.7N_1, \quad f_2 = 0.7N_2 \]

Solving we find \( F = 79 \text{ N}, \quad N_1 = 224.4 \text{ N}, \quad N_2 = 10.5 \text{ N} \)

Since \( N_2 > 0 \) we conclude that the table does not tip, and \( F = 79 \text{ N} \).
Problem 9.52  The coefficient of static friction between the right bar and the surface at $A$ is $\mu_s = 0.6$. Neglect the weights of the bars. If $\alpha = 20^\circ$, what is the magnitude of the friction force exerted at $A$?

Solution:  Note that the condition of impending slip does not necessarily apply. The moments about the left pin support:

$$\sum M = -F \cos \alpha + 2A \sin \alpha = 0,$$

from which $A = \frac{F}{2 \tan \alpha}$.

Isolate the right bar and take moments about the upper pin joint:

$$\sum M = AL \sin \alpha - fL \cos \alpha = 0,$$

from which $f = A \tan \alpha$. Substitute for $A$:

$$f = A \tan \alpha = \frac{F \tan \alpha}{2 \tan \alpha} = \frac{F}{2}$$

Problem 9.53  Consider the system shown in Problem 9.52. The coefficient of static friction between the right bar and the surface at $A$ is $\mu_s = 0.6$. Neglect the weight of the bars. What is the largest angle $\alpha$ at which the truss will remain stationary without slipping?

Solution:  From the solution to Problem 9.52, $f = A \tan \alpha$. Since $f = \mu_s A$ at impending slip, $\mu_s = \tan \alpha$, from which $\alpha = \tan^{-1}(0.6) = 30.96^\circ$. 

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Problem 9.54 The bar $BC$ is supported by a rough floor at $C$. If $F = 2$ kN and the bar $BC$ does not slip at $C$, what is the magnitude of the friction force exerted on the bar at $C$?

Solution: From the free-body diagram of bar $AB$,

$\Sigma M_A : B_y (1 \text{ m}) - F (0.5 \text{ m}) = 0,$

$\Rightarrow B_y = \frac{1}{2} F.$

From the free-body diagram of bar $BC$,

$\Sigma F_y : N - B_y = 0,$

$\Sigma M_B : N (0.2 \text{ m}) - f (0.6 \text{ m}) = 0$

We obtain

$f = \frac{1}{3} N = \frac{1}{2} B_y = \frac{1}{2} F = \frac{1}{2}(2000 \text{ N}).$

$f = 333 \text{ N}.$

Problem 9.55 The bar $BC$ is supported by a rough floor at $C$. If $F = 2$ kN what is the minimum coefficient of static friction for which bar $BC$ will not slip at $C$?

Solution: See the solution to Problem 9.54. Equilibrium requires that $f = \frac{1}{3} N$ independently of the value of $F$. If $\mu_s$ is less than $1/3$, the bar will slip for any value of $F$, and if $\mu_s$ is equal to or greater than $1/3$ the bar will not slip for any value of $F$.

$\mu_s = 1/3.$
Problem 9.56  The weight of the box is \( W = 20 \text{ N} \) and the coefficient of static friction between the box and the floor is \( \mu_s = 0.65 \). Neglect the weights of the bars. What is the largest value of the force \( F \) that will not cause the box to slip?

Solution:  Note that \( BC \) is a two force member. Member \( AB \)

\[
\sum F_x: \quad A_x - F_{BC} \cos 45^\circ = 0 \quad (1)
\]

\[
\sum F_y: \quad A_y - F + F_{BC} \sin 45^\circ = 0 \quad (2)
\]

\[
\sum M_A: \quad -4F + 8F_{BC} \sin 45^\circ = 0 \quad (3)
\]

Unknowns \( A_x, \ A_y, \ F, \ F_{BC} \)

\( W = 20 \text{ N} \)

\( \mu_s = 0.65 \)

\( f = \mu_s N \) for impending slip.

\[
\sum F_x: \quad \mu_s N + F_{BC} \cos 45^\circ = 0 \quad (4)
\]

\[
\sum F_y: \quad N - W - F_{BC} \sin 45^\circ = 0 \quad (5)
\]

Unknowns \( N, \ F_{BC}, \ A_x, \ A_y, \ F \) 5 eqns, 5 unknowns

Solving, we get

\( F = 74.3 \text{ N} \).
Problem 9.57 The mass of the suspended object is 6 kg. The structure is supported at B by the normal and friction forces exerted on the plate by the wall. Neglect the weights of the bars.

(a) What is the magnitude of the friction force exerted on the plate at B?
(b) What is the minimum coefficient of static friction at B necessary for the structure to remain in equilibrium?

Solution: Consider the weight as hanging from AB. Note that AB and AC are two force members. (m = 6 kg)

Joint B:
\[ \sum F_x: \quad N - F_{AB} \cos 8^\circ = 0 \]  
\[ \sum F_y: \quad f - F_{AB} \sin 8^\circ = 0 \]

Joint A:
\[ \sum F_x: \quad F_{AB} \cos 8^\circ - F_{AC} \cos 60^\circ = 0 \]  
\[ \sum F_y: \quad F_{AB} \sin 8^\circ + F_{AC} \sin 60^\circ - mg = 0 \]

Unknowns: \( F_{AB}, F_{AC}, N, f \)

Solving

(a) \( f = 4.42 \) N
\[ N = 31.43 \] N
\[ F_{AB} = 31.74 \] N
\[ F_{AC} = 62.86 \] N

(b) \( f = \mu_{MN} N \)
\[ \mu_{MN} = 0.141 \]
Problem 9.58  Suppose that the lengths of the bars in Problem 9.57 are $L_{AB} = 1.2$ m and $L_{AC} = 1.0$ m and their masses are $m_{AB} = 3.6$ kg and $m_{AC} = 3.0$ kg.

(a) What is the magnitude of the friction force exerted on the plate at $B$?
(b) What is the minimum coefficient of static friction at $B$ necessary for the structure to remain in equilibrium?

Solution: This problem differs greatly from Problem 9.57. Neither bar is a two force member. We must draw free-body diagrams of each bar and use both force and moment equilibrium in our solutions. Assume that the weight hangs from bar $AB$ and that the bars are uniform.

$m_L = 6$ kg

$m_{AB} = 3.6$ kg

\[ \sum F_x: N + A_x = 0 \quad (1) \]

\[ \sum F_y: f - m_{AB}g - m_Lg + A_y = 0 \quad (2) \]

\[ \sum M_B: - (0.6 \cos 8^\circ)m_{AB}g - (1.2 \sin 8^\circ)A_x + (1.2 \cos 8^\circ)(A_y - m_Lg) = 0 \quad (3) \]

$m_{AC} = 3.0$ kg

\[ \sum F_x: -A_x + C_x = 0 \quad (4) \]

\[ \sum F_y: C_y - A_y - m_{AC}g = 0 \quad (5) \]

\[ \sum M_A: (0.5 \cos 60^\circ)m_{AC}g - (1 \cos 60^\circ)C_y - (1 \sin 60^\circ)C_x = 0 \quad (6) \]

Unknowns:

$A_x, A_y, C_x, C_y, N, F$

We have 6 eqns. in 6 unknowns.

Solving, we get

(a) $f = 24.5$ N, $N = 48.7$ N

Also, $A_x = -48.7$ N, $A_y = 69.7$ N

$C_x = -48.7$ N, $C_y = 99.1$ N

(b) $\mu_{MIN} = \frac{f}{N} = 0.503$
Problem 9.59  The frame is supported by the normal and friction forces exerted on the plates at A and G by the fixed surfaces. The coefficient of static friction at A is $\mu_s = 0.6$. Will the frame slip at A when it is subjected to the loads shown?

Solution: The strategy is to write the equilibrium equations and solve for the unknown. The complete structure as a free body: Denote the normal forces as $A$ and $G$, and the friction forces as $f_A$ and $f_G$.

The sum of forces:
(1) $\sum F_x = -f_A + f_G + 8 = 0$.
(2) $\sum F_y = A + G - 6 = 0$.

The elements as free bodies: Element ABC: (See Figure)
(3) $\sum M_B = +4A + 2C_y - 12 = 0$.
(4) $\sum F_x = -f_A + C_x = 0$.
(5) $\sum F_y = A - B + C_y - 6 = 0$.

Element BE:
(6) $\sum F_y = B - E = 0$.

Element CD:
(7) $\sum M_C = +2D_x - D_y + 8 = 0$.
(8) $\sum F_x = -C_x + D_x + 8 = 0$.
(9) $\sum F_y = D_x - C_y = 0$.

Element DEG:
(10) $\sum M_D = -2G - E = 0$.
(11) $\sum F_x = f_G - D_x = 0$.
(12) $\sum F_y = G - D_y + E = 0$.

These twelve equations in ten unknowns can be solved by iteration or by back substitution. The results in detail:

$A = -24$ kN,
$f_A = 13$ kN,
$C_y = 18$ kN,
$C_x = 13$ kN,
$B = 36$ kN,
$D_x = 5$ kN,
$D_y = 18$ kN,
$E = 36$ kN,
$G = -18$ kN,
$f_G = 5$ kN.

The assumed directions are shown in the Figure; a negative sign means that the result is opposite to the assumed direction. The magnitude of the coefficient of static friction for the reaction at A required to hold the frame in equilibrium is

$\mu_{sA} = \frac{f_A}{A} = 0.5417$.

Since this is less than the known value, $\mu_s = 0.6$, the frame will not slip at A.
**Problem 9.60** The frame is supported by the normal and friction forces exerted on the plate at \( A \) by the wall.

(a) What is the magnitude of the friction force exerted on the plate at \( A \)?

(b) What is the minimum coefficient of static friction at \( A \) necessary for the structure to remain in equilibrium?

**Solution:** Draw a free body diagram of each member and write the corresponding equilibrium equations

\[
\sum F_x: \quad N + B_x = 0 \quad (1)
\]

\[
\sum F_y: \quad f + B_y + C_y = 0 \quad (2)
\]

\[
\sum M_A: \quad 2B_y + 4C_y = 0 \quad (3)
\]

\[
\sum F_x: \quad E_x + D_x - B_x = 0 \quad (4)
\]

\[
\sum F_y: \quad E_y + D_y - B_y = 0 \quad (5)
\]

\[
\sum M_E: \quad 1D_y + 1D_x - 2B_y - 2B_x = 0 \quad (6)
\]

\[
\sum F_x: \quad -D_x = 0 \quad (7)
\]

\[
\sum F_y: \quad -D_y - C_y - 6 = 0 \quad (8)
\]

\[
\sum M_B: \quad -3C_y - 4(6) = 0 \quad (9)
\]

**Unknowns:**

\( N, f, B_x, B_y, C_x, D_x, D_y, E_x, E_y \).

We have 9 eqns in 9 unknowns. Solving, we get

(a) \( f = -8 \text{ kN} \) (friction acts down)

Also \( N = 15 \text{ kN} \)

\( B_x = -15 \text{ kN}, \quad B_y = 16 \text{ kN} \)

\( C_y = -8 \text{ kN} \) (opposite the direction assumed)

\( D_x = 0, \quad D_y = 2 \text{ kN} \)

\( E_x = -15 \text{ kN}, \quad E_y = 14 \text{ kN} \)

(b) \( \mu_{\text{MIN}} = \frac{|f|}{N} = 0.533 \)
Problem 9.61  The direction cosines of the crane's cable are \( \cos \theta_x = 0.558, \cos \theta_y = 0.766, \cos \theta_z = 0.260 \). The \( y \) axis is vertical. The stationary caisson to which the cable is attached weighs 2000 lb and rests on horizontal ground. If the coefficient of static friction between the caisson and the ground is \( \mu_s = 0.4 \), what tension in the cable necessary to cause the caisson to slip?

Solution:  Let \( T \) be the force exerted on the caisson by the cable. We can express it in terms of the direction cosines as \( T = T(0.558i + 0.766j + 0.260k) \). If \( N = Nj \) is the normal force exerted on the caisson by the ground, the sum of the vertical forces on the caisson is \( \sum F_y = 0.766T + N - 2000 = 0 \).

1. The magnitude of the horizontal force exerted by the cable is \( \sqrt{(0.558)^2 + (0.260)^2} = 0.643T \). From the free-body diagram of the caisson, viewed perpendicular to the vertical plane containing the cable, we see that \( 0.643T - f = 0 \).

2. Slip impends when \( f = \mu_s N \).

3. Solving equation (1) for \( N \) and solving equation (2) for \( f \) and substituting the results into equation (3), we obtain \( 0.643T = \mu_s(2000 - 0.766T) \). The solution of this equation is \( T = 843 \text{ lb} \). 

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Problem 9.62* The 100 N metal disk \( A \) is at the center of the inclined surface. The tension in the string \( AB \) is 50 N. What minimum coefficient of static friction between the disk and the surface is necessary to keep the disk from slipping?

Solution: The coordinates of the disk \( A \) are \((5, 1, 4)\) m, so the position vector of pt. \( B \) relative to \( A \) is \( \mathbf{r}_{AB} = (0 - 5)\mathbf{i} + (6 - 1)\mathbf{j} + (0 - 4)\mathbf{k} \) m. We can express the force exerted on the disk by the string as

\[
\mathbf{T} = (50 \, \text{N}) \frac{\mathbf{r}_{AB}}{||\mathbf{r}_{AB}||} = -30.8\mathbf{i} + 30.8\mathbf{j} - 24.6\mathbf{k} \, \text{(N)}.
\]

The sum of the forces exerted on the disk by the string and the weight of the disk is

\[
\mathbf{T} - 100\mathbf{j} = -30.8\mathbf{i} - 69.2\mathbf{j} - 24.6\mathbf{k} \, \text{(N)}.
\]

The components of this force normal and parallel to the surface are balanced by the normal force and friction force, respectively. To determine these components, we need a unit vector \( \mathbf{e} \) perpendicular to the surface: The angle \( \beta = \arctan(2/8) = 14.0^\circ \), so

\[
\mathbf{e} = \cos \beta \mathbf{j} + \sin \beta \mathbf{k} = 0.970\mathbf{j} + 0.243\mathbf{k}.
\]

The component of \( \mathbf{T} - 100\mathbf{j} \) normal to the surface is

\[
[(\mathbf{T} - 100\mathbf{j}) \cdot \mathbf{e}]\mathbf{e} = -70.9\mathbf{j} - 17.7\mathbf{k} \, \text{(N)}.
\]

The magnitude of the normal force equals the magnitude of this vector:

\[
N = || -70.9\mathbf{j} - 17.7\mathbf{k} || = 73.1 \, \text{N}.
\]

The component of \( \mathbf{T} - 100\mathbf{j} \) parallel to the surface is

\[
[(\mathbf{T} - 100\mathbf{j}) - [(\mathbf{T} - 100\mathbf{j}) \cdot \mathbf{e}]\mathbf{e}] = -30.8\mathbf{i} + 1.7\mathbf{j} - 6.9\mathbf{k} \, \text{(N)}.
\]

The magnitude of the friction force is

\[
f = || -30.8\mathbf{i} + 1.7\mathbf{j} - 6.9\mathbf{k} || = 31.6 \, \text{N}.
\]

Slip impends when \( f = \mu s N \) so the minimum friction coefficient is

\[
\mu_s = \frac{f}{N} = \frac{31.6}{73.1} = 0.432.
\]
Problem 9.63* The 5-kg box is at rest on the sloping surface. The $y$ axis points upward. The unit vector $0.557\mathbf{i} + 0.743\mathbf{j} + 0.371\mathbf{k}$ is perpendicular to the sloping surface. What is the magnitude of the friction force exerted on the box by the surface?

Solution:

\[ N = N_{\text{mag}}(0.557\mathbf{i} + 0.743\mathbf{j} + 0.371\mathbf{k}) \]

\[ W = -(49.05\text{ N})\mathbf{j} \]

\[ \sum \mathbf{F} : \mathbf{f} + \mathbf{N} + \mathbf{W} = 0 \Rightarrow \mathbf{f} = -\mathbf{W} - \mathbf{N} \]

We also know that the friction force is parallel to the surface:

\[ \mathbf{f}(0.557\mathbf{i} + 0.743\mathbf{j} + 0.371\mathbf{k}) = 0 \Rightarrow N_{\text{mag}} = 36.45\text{ N} \]

Then the friction force is

\[ \mathbf{f} = (-20.3\mathbf{i} + 22.0\mathbf{j} - 13.5\mathbf{k})\text{ N} \Rightarrow f = 32.83\text{ N} \]

Problem 9.64* In Problem 9.63, what is the minimum coefficient of static friction necessary for the box to remain at rest on the sloping surface?

Solution: See 9.63

\[ \mu_s = \frac{f}{N_{\text{mag}}} = \frac{32.83\text{ N}}{36.45\text{ N}} = 0.901 \]

Problem 9.65 In Active Example 9.4, the coefficients of friction between the wedge and the log are $\mu_s = 0.22$ and $\mu_k = 0.20$. What is the largest value of the wedge angle $\alpha$ for which the wedge would remain in place in the log when the force $F$ is removed?

Solution: Assume that $F = 0$ and the wedge is on the verge of slipping out of the log. The sum of the forces in the vertical direction is

\[ \sum F_y : 2N \sin \left( \frac{\alpha}{2} \right) - 2\mu_s N \cos \left( \frac{\alpha}{2} \right) = 0. \]

The wedge will not slip out if

\[ \mu_s \geq \tan \left( \frac{\alpha}{2} \right) \Rightarrow \alpha \leq 2 \tan^{-1}(\mu_s). \]

If $\mu_s = 0.22$, the largest value of $\alpha$ for which the wedge will remain in place is

\[ \alpha = 24.8^\circ. \]
**Problem 9.66** The wedge shown is being used to split the log. The wedge weighs 100 N and the angle $\alpha$ equals 30°. The coefficient of kinetic friction between the faces of the wedge and the log is 0.28. If the normal force exerted by each face of the wedge must equal 600 N to split the log, what vertical force $F$ is necessary to drive the wedge into the log at a constant rate? (See Active Example 9.4.)

**Solution:** The free-body diagram is shown.

Summing forces in the vertical direction gives

$$\sum F_y = 2N \sin \left(\frac{\alpha}{2}\right) + 2\mu_k N \cos \left(\frac{\alpha}{2}\right) - W - F = 0$$

Thus

$$F = 2N \left[ \sin \left(\frac{\alpha}{2}\right) + \mu_k \cos \left(\frac{\alpha}{2}\right) \right] - W$$

$$= 2(600 \text{ N}) \left[ \sin 15^\circ + (0.28) \cos 15^\circ \right] - 100 \text{ N}$$

$$= 535 \text{ N}$$

$$F = 535 \text{ N}.$$ 

---

**Problem 9.67** The coefficient of static friction between the faces of the wedge and the log in Problem 9.66 is 0.30. Will the wedge remain in place in the log when the vertical force $F$ is removed? (See Active Example 9.4.)

**Solution:** Assume that $F = 0$ and the wedge is on the verge of slipping out of the log. The sum of the forces in the vertical direction is

$$\sum F_y : 2N \sin \left(\frac{\alpha}{2}\right) - 2\mu_s N \cos \left(\frac{\alpha}{2}\right) = 0.$$ 

The wedge will not slip out if

$$\mu_s \geq \tan \left(\frac{\alpha}{2}\right) = \tan 15^\circ$$

$$0.3 > 0.268 \Rightarrow yes.$$
Problem 9.68 The weights of the blocks are \( W_A = 100 \text{ N} \) and \( W_B = 25 \text{ N} \). Between all of the contacting surfaces, \( \mu_s = 0.32 \) and \( \mu_k = 0.30 \). What force \( F \) is necessary to move \( B \) to the left at a constant rate?

Solution: We have 7 unknowns, 4 equilibrium equations and 3 friction equations:

\[
\sum F_{Ax} : N_1 - f_2 \cos 10^\circ - N_2 \sin 10^\circ = 0
\]

\[
\sum F_{Ay} : -f_1 + N_2 \cos 10^\circ - f_2 \sin 10^\circ - 100 = 0
\]

\[
\sum F_{Bx} : f_2 \cos 10^\circ + N_2 \sin 10^\circ + f_3 - F = 0
\]

\[
\sum F_{By} : f_2 \sin 10^\circ - N_2 \cos 10^\circ + N_3 - 25 = 0
\]

\( f_1 = 0.3N_1, \quad f_2 = 0.3N_2, \quad f_3 = 0.3N_3 \)

Solving we find \( F = 102 \text{ N} \).
Problem 9.69 The masses of the blocks are \( m_A = 30 \, \text{kg} \) and \( m_B = 70 \, \text{kg} \). Between all of the contacting surfaces, \( \mu_s = 0.1 \). What is the largest force \( F \) that can be applied without causing the blocks to slip?

Solution: The equilibrium equations for block \( A \) are

\[
\Sigma F_x : R - F \sin 30^\circ + \mu_s P \cos 20^\circ = 0,
\]

\[
\Sigma F_y : -F \sin 20^\circ + \mu_s R - m_A g = 0.
\]

The equilibrium equations for block \( B \) are

\[
\Sigma F_x : P \cos 20^\circ - \mu_s P \cos 20^\circ = 0,
\]

\[
\Sigma F_y : -P \cos 20^\circ + \mu_s P \sin 20^\circ - m_B g + N = 0.
\]

Substituting the given values and solving yields

\( R = 212 \, \text{N} \), \( P = 456 \, \text{N} \), \( N = 1130 \, \text{N} \), and \( F = 197 \, \text{N} \).

\( F = 197 \, \text{N} \).
Problem 9.70 Each block weighs 200 N. Between all of the contacting surfaces, $\mu_s = 0.1$. What is the largest force $F$ that can be applied without causing block $B$ to slip upward?

Solution: Let $W = 200$ N.

The equilibrium equations for block $B$ are

$$\sum F_x : R \cos 10^\circ + \mu_s R \sin 10^\circ = 0,$$

$$\sum F_y : R \sin 10^\circ - \mu_s R \cos 10^\circ = 0,$$

$$+ P \sin 10^\circ - \mu_s P \cos 10^\circ - W = 0.$$

The equilibrium equations for block $C$ are

$$\sum F_x : P \cos 10^\circ + \mu_s P \sin 10^\circ = 0,$$

$$+ \mu_s N - F = 0,$$

$$\sum F_y : N + \mu_s P \cos 10^\circ = 0,$$

$$- P \sin 10^\circ - W = 0.$$

Substituting the given values and solving yields

$P = R = 1330$ N, $N = 300$ N, and $F = 1360$ N.

$F = 1360$ N.
**Problem 9.71** Small wedges called *shims* can be used to hold an object in place. The coefficient of kinetic friction between the contacting surfaces is 0.4. What force $F$ is needed to push the shim downward until the horizontal force exerted on the object $A$ is 200 N?

![Diagram of a Shim and Object]

**Solution:**

1. $f_L = \mu_k N_L$  \hspace{1cm} (1)
2. $f_R = \mu_k N_R$  \hspace{1cm} (2)
3. $N_L = 200 \text{ N}$  \hspace{1cm} (3)
4. $\sum F_x: N_L - N_R \cos 5^\circ + f_R \sin 5^\circ = 0$  \hspace{1cm} (4)
5. $\sum F_y: -F + f_L + f_R \cos 5^\circ + N_R \sin 5^\circ = 0$  \hspace{1cm} (5)

Unknowns: $f_L, f_R, N_L, N_R, F$

Solving,

$F = 181 \text{ N}$

**Problem 9.72** The coefficient of static friction between the contacting surfaces in Problem 9.71 is 0.44. If the shims are in place and exert a 200-N horizontal force on the object $A$, what upward force must be exerted on the left shim to loosen it?

![Diagram of a Shim and Object]

**Solution:**

1. $F_L = \mu_s N_L$  \hspace{1cm} (1)
2. $F_R = \mu_s N_R$  \hspace{1cm} (2)
3. $\mu_s = 0.44$
4. $N_L = 200 \text{ N}$
5. $\sum F_x: N_L - N_R \cos 5^\circ - f_R \sin 5^\circ = 0$  \hspace{1cm} (3)
6. $\sum F_y: -F + f_L + F_R \cos 5^\circ + N_R \sin 5^\circ = 0$  \hspace{1cm} (4)

Unknowns $F, f_L, f_R, N_R$

Solving, $F = 156 \text{ N}$
Problem 9.73  The crate $A$ weighs 600 N. Between all contacting surfaces, $\mu_s = 0.32$ and $\mu_k = 0.30$. Neglect the weights of the wedges. What force $F$ is required to move $A$ to the right at a constant rate?

Solution:  The active sliding contact surfaces are between the wall and the left wedge, between the wedges, between the floor and the bottom of the right wedge, and between the crate and the floor. Leftmost wedge: Denote the normal force exerted by the wall by $Q$, and the normal force between the wedges by $N$. The equilibrium conditions for the left wedge moving at a constant rate are:

$$\sum F_y = -F + \mu_s N \cos \alpha + N \sin \alpha + \mu_k Q = 0.$$  
$$\sum F_x = Q - N \cos \alpha + \mu_k N \sin \alpha = 0.$$  

For the right wedge: Denote the normal force exerted by the crate by $A$, and the normal force exerted by the floor by $P$.

$$\sum F_y = -N \sin \alpha - \mu_k N \cos \alpha + P = 0.$$  
$$\sum F_x = N \cos \alpha - \mu_k N \sin \alpha - \mu_k P - A = 0.$$  

For the crate: Denote the weight of the crate by $W$.

$$\sum F_x = A - \mu_k W = 0.$$  

These five equations are solved for the five unknowns by iteration:

$Q = 204.4$ N,
$N = 210.7$ N,
$P = 81.34$ N,
$A = 180$ N,
and $F = 142.66$ N.
Problem 9.74  Suppose that between all contacting surfaces in Problem 9.73, \( \mu_S = 0.32 \) and \( \mu_k = 0.30 \). Neglect the weights of the 5° wedges. If a force \( F = 800 \text{ N} \) is required to move \( A \) to the right at a constant rate, what is the mass of \( A \)?

Solution:  The free body diagrams of the left wedge and the combined right wedge and crate are as shown. The equilibrium equations are

Wedge:
\[
\sum F_x = N - P \cos 5° + 0.3P \sin 5° = 0,
\]
\[
\sum F_y = 0.3N + P \sin 5° + 0.3P \cos 5° - F = 0,
\]

Wedge and box:
\[
\sum F_x = P \cos 5° - 0.3P \sin 5° - 0.3Q = 0,
\]
\[
\sum F_y = Q - P \sin 5° - 0.3P \cos 5° - 9.81 \text{ m} = 0.
\]

Solving them, we obtain

\[ P = 1180 \text{ N}, \]
\[ N = 1150 \text{ N}, \]
\[ Q = 3820 \text{ N}, \]
and \( m = 343 \text{ kg} \).

Problem 9.75  The box \( A \) has a mass of 80 kg, and the wedge \( B \) has a mass of 40 kg. Between all contacting surfaces, \( \mu_S = 0.15 \) and \( \mu_k = 0.12 \). What force \( F \) is required to raise \( A \) at a constant rate?

Solution:  From the free-body diagrams shown, the equilibrium equations are

Box \( A \):
\[
\sum F_x = 0:
\]
\[
Q - N \sin 10° - \mu_k N \cos 10° = 0,
\]
\[
\sum F_y = 0:
\]
\[
N \cos 10° - \mu_k N \sin 10° - \mu_k Q - W = 0.
\]

Wedge \( B \):
\[
\sum F_x = 0:
\]
\[
P \sin 10° + \mu_k P \cos 10° + N \sin 10° + \mu_k N \cos 10° - F = 0,
\]
\[
\sum F_y = 0:
\]
\[
P \cos 10° - \mu_k P \sin 10° - N \cos 10° + \mu_k N \sin 10° - W_w = 0.
\]

Solving with
\[
W = (80)(9.81) \text{ N},
\]
\[
W_w = (40)(9.81) \text{ N},
\]
and \( \mu_k = 0.12 \),

we obtain

\[ N = 845 \text{ N}, \]
\[ Q = 247 \text{ N}, \]
\[ P = 1252 \text{ N}, \]
and \( F = 612 \text{ N} \).
Problem 9.76  Suppose that in Problem 9.75, A weighs 800 N and B weighs 400 N. The coefficients of friction between all of the contacting surfaces are $\mu_A = 0.15$ and $\mu_B = 0.12$. Will B remain in place if the force $F$ is removed?

**Solution:** The equilibrium conditions are: For the box A: Denote the normal force exerted by the wall by $Q$, and the normal force exerted by the wedge by $N$. The friction forces oppose motion.

\[
\sum F_x = -W + N \cos \alpha + \mu_N \sin \alpha + \mu_P Q = 0,
\]
\[
\sum F_y = +\mu_P N \cos \alpha - N \sin \alpha + Q = 0.
\]

For the wedge B: Denote the normal force on the lower surface by $P$.

\[
\sum F_x = -\mu_B N \cos \alpha - \mu_B P \cos \alpha + P \sin \alpha + N \sin \alpha = 0.
\]
\[
\sum F_y = -N \cos \alpha + P \cos \alpha - \mu_B N \sin \alpha + \mu_B P \sin \alpha - W = 0.
\]

(A comparison with the equilibrium conditions for Problem 9.75 will show that the friction forces are reversed, since for slippage the box A will move downward, and the wedge B to the right.) The strategy is to solve these equations for the required $\mu_B$ to keep the wedge B in place when $F = 0$. The solution $Q = 0$, $N = 787.8$ N, $P = 1181.8$ N and $\mu_B = 0.1763$. Since the value of $\mu_B$ required to hold the wedge in place is greater than the value given, the wedge will slip out.

![Diagram](image)

Problem 9.77  Between A and B, $\mu_B = 0.20$, and between B and C, $\mu_C = 0.18$. Between C and the wall, $\mu_S = 0.30$. The weights $W_B = 20$ N and $W_C = 80$ N. What force $F$ is required to start C moving upward?

**Solution:** The active contact surfaces are between the wall and C, between the wedge B and C, and between the wedge B and A. For the weights C: Denote the normal force exerted by the wall by $Q$, and the normal force between B and C by $N$. Denote the several coefficients of static friction by subscripts. The equilibrium conditions are:

\[
\sum F_x = -W_C + N - \mu_C W_C Q = 0,
\]
\[
\sum F_y = -Q + \mu_C N = 0.
\]

For the wedge B: Denote the normal force between A and B by $P$.

\[
\sum F_x = -N + P \cos \alpha - \mu_B P \sin \alpha - W_B = 0.
\]
\[
\sum F_y = F - \mu_B N - \mu_B P \cos \alpha - P \sin \alpha = 0.
\]

These four equations in four unknowns are solved:

\[
Q = 15.2 \text{ N},
\]
\[
N = 84.6 \text{ N},
\]
\[
F = 114.4 \text{ N},
\]
\[
and \quad F = 66.9 \text{ N}.
\]
**Problem 9.78** The masses of $A$, $B$, and $C$ are 8 kg, 12 kg, and 80 kg, respectively. Between all contacting surfaces, $\mu_s = 0.4$. What force $F$ is required to start $C$ moving upward?

**Solution:** The active contact surfaces are between $A$ and $B$, between $A$ and the wall, between $B$ and the floor, and between $B$ and $C$. Assume that the roller supports between $C$ and the wall exert no friction forces.

For the wedge $A$: Denote the normal force exerted by the wall as $Q$ and the normal force between $A$ and $B$ as $N$. The weight is $W_A = 8 \, \text{g} = 78.48 \, \text{N}$. The equilibrium conditions:

\[
\sum F_y = -F + \mu_s Q + \mu_s N \cos \alpha + N \sin \alpha - W_A = 0
\]
\[
\sum F_y = Q - N \cos \alpha + \mu_s N \sin \alpha = 0.
\]

For wedge $B$: Denote the normal force exerted on $B$ by the floor by $P$, and the normal exerted by the weight $C$ as $S$. The weight of $B$ is $W_B = 12 \, \text{g} = 117.72 \, \text{N}$. The equilibrium conditions:

\[
\sum F_y = -N \sin \alpha - S \cos \beta + P - \mu_s N \cos \alpha + \mu_s S \sin \beta - W_B
\]
\[
= 0.
\]
\[
\sum F_x = N \cos \alpha - \mu_s N \sin \alpha - \mu_s P - \mu_s S \cos \beta - S \sin \beta = 0.
\]

For the weight $C$: The weight is $W_C = 80 \, \text{g} = 784.8 \, \text{N}$. The equilibrium conditions:

\[
\sum F_y = -W_C + S \cos \beta - \mu_s S \sin \beta = 0.
\]

These five equations in five unknowns are solved:

\[
Q = 1157.6 \, \text{N},
\]
\[
N = 1293.5 \, \text{N},
\]
\[
S = 857.4 \, \text{N},
\]
\[
P = 1677.5,
\]
and $F = 1160 \, \text{N}$.
Problem 9.79  In Active Example 9.5, suppose that the pitch of the thread is changed from \( p \) = 0.2 cm to \( p \) = 0.24 cm. What is the slope of the thread? What is the magnitude of the couple that must be applied to the collar \( C \) to cause it to turn at a constant rate and move the suspended object upward?

**Solution:** The slope \( \alpha \) is determined from the relation

\[
\tan \alpha = \frac{p}{2\pi r} = \frac{0.24 \text{ cm}}{2\pi(1.6 \text{ cm})} = 0.0239
\]

\( \Rightarrow \alpha = 1.37^\circ \).

Using this value, the required couple is

\[
M = rF \tan(\theta_k + \alpha) = (1.6 \text{ cm})(200 \text{ N}) \tan(12.4^\circ + 1.37^\circ) = 78.5 \text{ cm}
\]

\[
M = 78.5 \text{ N-cm}
\]

Problem 9.80  Suppose that in Problem 9.79, the pitch of the threaded shaft is \( p \) = 2 mm and the mean radius of the thread is \( r \) = 20 mm. The coefficients of friction between the thread and the mating groove are \( \mu_k = 0.22 \), and \( \mu_k = 0.20 \). The weight \( W \) = 500 N. Neglect the weight of the threaded shaft. What couple must be applied to the threaded shaft to lower the weight at a constant rate?

**Solution:** The angle of kinetic friction is

\[
\theta_k = \tan^{-1}(0.2) = 11.31^\circ.
\]

The angle of pitch is

\[
\alpha = \tan^{-1} \left( \frac{p}{2\pi r} \right) = \tan^{-1} \left( \frac{2}{2\pi(20)} \right) = 0.9118^\circ.
\]

The moment required to lower the weight at a constant rate is

\[
M = 0.02(500) \tan(11.31^\circ - 0.9118^\circ) = 1.835 \text{ N-m}.
\]
Problem 9.81 The position of the horizontal beam can be adjusted by turning the machine screw A. Neglect the weight of the beam. The pitch of the screw is $p = 1\ \text{mm}$, and the mean radius of the thread is $r = 4\ \text{mm}$. The coefficients of friction between the thread and the mating groove are $\mu_s = 0.20$ and $\mu_k = 0.18$. If the system is initially stationary, determine the couple that must be applied to the screw to cause the beam to start moving (a) upward; (b) downward.

Solution: The sum of the moments about the pin support is

$$\sum M = -0.4F + (0.3)400 = 0,$$

from which the force exerted by the screw is $F = 300\ \text{N}$. The pitch angle is

$$\alpha = \tan^{-1}\left(\frac{1}{2\pi(4)}\right) = 2.28^\circ.$$

The static friction angle is $\theta_s = \tan^{-1}(0.2) = 11.31^\circ$. (a) The moment required to start motion upward is

$$M = 0.004(300)\tan(11.31^\circ + 2.28^\circ) = 0.29\ \text{N}\cdot\text{m}$$

(b) The moment required to start motion downward is

$$M = 0.004(300)\tan(11.31^\circ - 2.28^\circ) = 0.19\ \text{N}\cdot\text{m}$$

Problem 9.82 The pitch of the threaded shaft of the C clamp is $p = 1\ \text{mm}$ and the mean radius of the thread is $r = 3\ \text{mm}$. The coefficients of friction between the threaded shaft and the mating collar are $\mu_s = 0.18$ and $\mu_k = 0.16$.

(a) What maximum couple must be applied to the shaft to exert a 30-N force on the clamped object?
(b) If a 30-N force is exerted on the clamped object, what couple must be applied to the shaft to begin loosening the clamp?

Solution:

$F = 30\ \text{N},\ r = 0.003\ \text{m},\ p = 0.001\ \text{m},\ \mu_s = 0.18,\ \mu_k = 0.16$

$\theta_s = \tan^{-1}(\mu_s) = 10.2^\circ,\ \theta_k = \tan^{-1}(0.16) = 9.09^\circ$

$\alpha = \tan^{-1}\left(\frac{p}{2\pi r}\right) = 3.04^\circ$

(a) $M = rF\tan(\theta_s + \alpha) = 0.01934\ \text{N}\cdot\text{m}$

(b) $M = rF\tan(\theta_s - \alpha) = 0.01132\ \text{N}\cdot\text{m}$
Problem 9.83  The mass of block $A$ is 60 kg. Neglect the weight of the $5^\circ$ wedge. The coefficient of kinetic friction between the contacting surfaces of the block $A$, the wedge, the table, and the wall is $\mu_k = 0.4$. The pitch of the threaded shaft is 5 mm, the mean radius of the thread is 15 mm, and the coefficient of kinetic friction between the thread and the mating groove is 0.2. What couple must be exerted on the threaded shaft to raise the block $A$ at a constant rate?

Solution:  Denote the wedge angle by $\beta = 5^\circ$ and the normal force on the top by $N$ and on the lower surface by $P$. The free body diagrams of the wedge and block are as shown. The equilibrium equations for wedge:

$$\sum F_x = F - \mu_k P - N \sin 5^\circ - \mu_k N \cos 5^\circ = 0,$$
$$\sum F_y = P - N \cos 5^\circ + \mu_k N \sin 5^\circ = 0.$$

For the Block:

$$\sum F_x = N \sin 5^\circ + \mu_k N \cos 5^\circ - Q = 0,$$
$$\sum F_y = N \cos 5^\circ - \mu_k N \sin 5^\circ - \mu_k Q - W = 0.$$

Solving them, we obtain $F = 668$ N. From Equation (9.9), the couple necessary to rotate the threaded shaft when it is subjected to the axial force $F$ is $M = rP \tan(\theta_k + \alpha)$, $r$ is the radius 15 mm = 0.015 m, $\theta_k$ is the angle of kinetic friction $\theta_k = \arctan(0.2) = 11.31^\circ$.

From Equation (9.7), the slope is given in terms of the pitch by

$$\alpha = \arctan \left( \frac{P}{2 \pi r} \right) = \arctan \left( \frac{5}{2 \pi (15)} \right) = 3.04^\circ.$$

The couple is

$$M = (0.015 \text{ m})(668 \text{ N}) \tan(11.31^\circ + 3.04^\circ) = 2.56 \text{ N-m}.$$
Problem 9.84  The vise exerts 80-N forces on \( A \). The threaded shafts are subjected only to axial loads by the jaws of the vise. The pitch of their threads is \( p = 3 \text{ mm} \), the mean radius of the threads is \( r = 1 \text{ mm} \), and the coefficient of static friction between the threads and the mating grooves is \( 0.2 \). Suppose that you want to loosen the vise by turning one of the shafts. Determine the couple you must apply (a) to shaft \( B \); (b) to shaft \( C \).

Solution:  Isolate the left jaw. The sum of the moments about \( C \):

\[
\sum M_C = -0.096 B + 0.192 (80) = 0,
\]

from which \( B = 160 \text{ N} \). The sum of the forces:

\[
\sum F_x = -80 + B - C = 0,
\]

from which \( C = 80 \text{ N} \). The pitch angle is

\[
\alpha = \tan^{-1} \left( \frac{1}{16r} \right) = 1.14^\circ.
\]

The static friction angle is \( \phi_s = \tan^{-1}(0.2) = 11.31^\circ \). The moments required to loosen the vise are

\[
M_B = (0.024)(160)\tan(11.31^\circ - 1.14^\circ) = 0.69 \text{ N-m},
\]

and

\[
M_C = rC\tan(\phi_s - \alpha) = 0.34 \text{ N-m}.
\]

Problem 9.85  Suppose that you want to tighten the vise in Problem 9.84 by turning one of the shafts. Determine the couple you must apply (a) to shaft \( B \); (b) to shaft \( C \).

Solution:  Use the solution to Problem 9.84. (a) The moment on shaft \( B \) required to tighten the vise is \( M_B = rN\tan(\phi_s + \alpha) \). Note that

\[
r = 0.024, \quad B = 160 \text{ N},
\]

\[
\alpha = \tan^{-1} \left( \frac{1}{16r} \right) = 1.14^\circ,
\]

and

\[
\phi_s = \tan^{-1}(0.2) = 11.31^\circ,
\]

then

\[
M_B = 0.85 \text{ N-m}
\]

(b) For shaft \( C \), \( M_C = rC\tan(\phi_s + \alpha) \), where \( C = 80 \text{ N} \), \( M_C = 0.424 \text{ N-m} \).
Problem 9.86  The threaded shaft has a ball and socket support at A. The 400-N load A can be raised or lowered by rotating the threaded shaft, causing the threaded collar at C to move relative to the shaft. Neglect the weights of the members. The pitch of the shaft is \( p = 10 \text{ mm} \), the mean radius of the thread is \( r = 2.5 \text{ mm} \), and the coefficient of static friction between the thread and the mating groove is 0.24. If the system is stationary in the position shown, what couple is necessary to start the shaft rotating to raise the load?

Solution:  Denote the lower right pin support by D. The length of the connecting member \( CD \) is \( L_{CD} = \sqrt{9^2 + 12^2} = 15 \text{ cm} \). The angle between the threaded shaft and member \( CD \) is

\[
\beta = 2 \tan^{-1} \left( \frac{9}{12} \right) = 73.74^\circ.
\]

The sum of the moments about \( D \) is

\[
\sum M_D = L_{CD} F \cos(90 - \beta) - 0.18 W = 0,
\]

from which \( F = 500 \text{ N} \). The pitch angle is

\[
\alpha = \tan^{-1} \left( \frac{p}{2r} \right) = 2.28^\circ.
\]

The angle of static friction is \( \theta_s = \tan^{-1}(0.24) = 13.5^\circ \). The moment needed to start the threaded collar in motion is

\[
M = r F \tan(\theta_s + \alpha) = (0.01)(500) \tan(13.5^\circ + 2.28^\circ) = 1.412 \text{ N-m}
\]

Problem 9.87  In Problem 9.86, if the system is stationary in the position shown, what couple is necessary to start the shaft rotating to lower the load?

Solution:  Use the results of the solution to Problem 9.86. The moment is \( M = r F \tan(\theta_s + \alpha) \), where

\[
r = 0.01 \text{ m},
\]

\[
F = 500 \text{ N},
\]

\[
\theta_s = 13.5^\circ,
\]

and \( \alpha = 2.28^\circ \),

from which \( M = 0.992 \text{ N-m} \).
Problem 9.88  The car jack is operated by turning the horizontal threaded shaft at \( A \). The threaded shaft fits into a mating threading collar at \( B \). As the shaft turns, points \( A \) and \( B \) move closer together or farther apart, thereby raising or lowering the jack. The pitch of the threaded shaft is \( p = 2.5 \text{ mm} \), the mean radius of the thread is \( r = 5.5 \text{ mm} \), and the coefficient of kinetic friction between the threaded shaft and the mating collar at \( B \) is 0.15. What couple must be applied at \( A \) to rotate the shaft at a constant rate and raise the jack when it is in the position shown if the load \( L = 6500 \text{ N} \)?

**Solution:** Note that \( \beta = \tan^{-1}(65/120) = 28.4^\circ \).

Because of symmetry we know that
\[
F_{BC} = F_{AC} = F_{BD}.
\]
The equilibrium equations are
\[
\Sigma F_y : -L - 2F_{BC} \sin \beta = 0
\]
\[
\Sigma F_x : 2F_{BC} \cos \beta + F = 0
\]
Solving yields
\[
F = L \cot \beta
\]
\[
= (6500 \text{ N}) \cot(28.4^\circ) = 12021 \text{ N}.
\]
The angle of kinetic friction is
\[
\theta_k = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = 8.53^\circ.
\]
The slope of the thread is
\[
\alpha = \tan^{-1} \left( \frac{p}{2\pi r} \right) = \tan^{-1} \left[ \frac{2.5 \text{ mm}}{2\pi(5.5 \text{ mm})} \right] = 4.14^\circ.
\]
Therefore, the required couple is
\[
M = rF \tan(\theta_k + \alpha) = (5.5 \text{ mm})(12021 \text{ N}) \tan(8.53^\circ + 4.14^\circ) = 14863 \text{ N-mm}.
\]
\[
M = 14863 \text{ N-mm}.
\]
Problem 9.89 The car jack is operated by turning the horizontal threaded shaft at A. The threaded shaft fits into a mating threading collar at B. As the shaft turns, points A and B move closer together or farther apart, thereby raising or lowering the jack. The pitch of the threaded shaft is \( p \). The coefficient of kinetic friction between the threaded shaft and the mating collar at B is 0.15. What couple must be applied at A to rotate the shaft at a constant rate and lower the jack when it is in the position shown if the load \( L = 6500 \) N?

Solution: Note that \( \beta = \tan^{-1}(65/120) = 28.4^\circ \). Because of symmetry we know that 
\[ F_{BC} = F_{AC} = F_{BD}. \]

The equilibrium equations are
\[ \Sigma F_y = -L - 2F_{BC} \sin \beta = 0 \]
\[ \Sigma F_x = 2F_{AC} \cos \beta + F = 0 \]
Solving yields
\[ F = L \cot \beta \]
\[ = (6500 \text{ N} \cot(28.4^\circ)) = 12021 \text{ N}. \]

The angle of kinetic friction is
\[ \theta_k = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = 8.53^\circ. \]

The slope of the threads is
\[ \alpha = \tan^{-1} \left( \frac{p}{2\pi r} \right) = \tan^{-1} \left[ \frac{2.5 \text{ mm}}{2\pi(5.5 \text{ mm})} \right] = 4.14^\circ. \]

Therefore, the required couple is
\[ M = r F \tan(\theta_k - \alpha) = (5.5 \text{ mm})(12021 \text{ N}) \tan(8.53^\circ - 4.14^\circ) = 5076 \text{ N-mm}. \]

\[ M = 5076 \text{ N-mm}. \]

Problem 9.90 A turnbuckle, used to adjust the length or tension of a bar or cable, is threaded at both ends. Rotating it draws threaded ends of the bar or cable together or moves them apart. Suppose that the pitch of the threads is \( p = 3 \) mm, their mean radius is \( r = 25 \) mm, and the coefficient of static friction between the threads and the mating grooves is 0.24. If \( T = 800 \) N, what couple must be exerted on the turnbuckle to start tightening it?

Solution: The slope of the threads is
\[ \alpha = \tan^{-1} \left( \frac{p}{2\pi r} \right) = \tan^{-1} \left[ \frac{3 \text{ mm}}{2\pi(25 \text{ mm})} \right] = 1.09^\circ. \]

The angle of static friction is \( \theta_k = \tan^{-1}(\mu_k) = \tan^{-1}(0.24) = 13.5^\circ. \)

Using these values, one half of the required couple is
\[ M = r F \tan(\theta_k + \alpha) = (25 \text{ mm})(800 \text{ N}) \tan(13.5^\circ + 1.09^\circ) = 5206 \text{ N-mm}. \]

The required couple is
\[ 2M = 2(5206) = 10412 \text{ N-mm}. \]

\[ 10412 \text{ N-mm} \]
Problem 9.91 Suppose that the pitch of the threads of the turnbuckle is \( p = 3 \text{ mm} \), their mean radius is \( r = 25 \text{ mm} \), and the coefficient of static friction between the threads and the mating grooves is 0.24. If \( T = 800 \text{ N} \), what couple must be exerted on the turnbuckle to start loosening it?

Solution: The slope of the threads is

\[ \alpha = \tan^{-1} \left( \frac{p}{2\pi r} \right) = \tan^{-1} \left( \frac{3 \text{ mm}}{2\pi (25 \text{ mm})} \right) = 1.09^\circ. \]

The angle of static friction is \( \theta_0 = \tan^{-1}(\mu_s) = \tan^{-1}(0.24) = 13.5^\circ. \)
Using these values, one half of the required couple is

\[ M = rF \tan(\theta_0 - \alpha) = (25 \text{ mm})(800 \text{ N}) \tan(13.5^\circ - 1.09^\circ) = 4401 \text{ N-mm}. \]

The required couple is \( 2M = 2(4401 \text{ N-mm}) = 8802 \text{ N-mm}. \)
Problem 9.92 Member $BE$ of the frame has a turn-buckle. (See Problem 9.90.) The threads have pitch $p = 1$ mm, their mean radius is $r = 6$ mm, and the coefficient of static friction between the threads and the mating grooves is $0.2$. What couple must be exerted on the turn-buckle to start loosening it?

**Solution:** This problem has two parts. First, we find the tension in the two force member $BE$. Then we analyze the turnbuckle.

\[
\tan \theta = \frac{0.5}{0.4} \quad \theta = 51.3' \]
\[
\tan \phi = \frac{0.5}{0.2} \quad \phi = 68.2' \]

\[
\sum F_x: \quad A_x + T_{BE} \cos \theta + F_{CF} \cos \phi = 0 \quad (1)
\]
\[
\sum F_y: \quad A_y - T_{BE} \sin \theta + F_{CF} \sin \phi - 600 = 0 \quad (2)
\]
\[
\sum M_A: \quad -0.4T_{BE} \sin \theta + 1.4F_{CF} \sin \phi - 1.4(600) = 0 \quad (3)
\]
\[
\sum F_x: \quad D_x - T_{BE} \cos \theta - F_{CF} \cos \phi = 0 \quad (4)
\]
\[
\sum F_y: \quad D_y + T_{BE} \sin \theta - F_{CF} \sin \phi = 0 \quad (5)
\]
\[
\sum M_D: \quad 0.8T_{BE} \sin \theta - 1.2F_{CF} \sin \phi = 0 \quad (6)
\]

Unknowns: $A_x, A_y, D_x, D_y, T_{BE}, F_{CF}$ (6 eqns, 6 unknowns)

Solving,

\[ T_{BE} = 2017 \text{ N} \]

Now to analyze the turnbuckle

\[
\tan \theta_s = \mu_s = 0.2 \quad \theta_s = 11.31' \]
\[
\tan \alpha = \frac{p}{2r} = \frac{1}{2 \pi(0)} \quad \alpha = 1.52' \]

For one screw, to loosen

\[ M = rT_{BE} \tan(\theta_s - \alpha) \]
\[ M = 2.09 \text{ N} \cdot \text{m} \]

For two screws (turnbuckle)

\[ M_{\text{TOTAL}} = 4.18 \text{ N} \cdot \text{m} \]
Problem 9.93 In Problem 9.92, what couple must be exerted on the turnbuckle to start tightening it?

Solution: In Problem 9.92, the tension in the turnbuckle was

\[ T_{BE} = 2017 \text{ N} \]
\[ r = 0.006 \text{ m} \]
\[ p = 0.001 \text{ mm} \]
\[ \tan \theta_s = \mu_s = 0.2 \]
\[ \theta_s = 11.31^\circ \]
\[ \tan \alpha = \frac{p}{2\pi r} \]
\[ \alpha = 1.52^\circ \]

For one screw, to tighten,

\[ M = rT_{BE} \tan (\theta_s + \alpha) \]
\[ M = 2.756 \text{ N-m} \]

For two screws (turnbuckle)

\[ M = 5.51 \text{ N-m} \]
Problem 9.94 Members CD and DG of the truss have turnbuckles. (See Problem 9.90.) The pitch of the threads is \( p = 4 \) mm, their mean radius is \( r = 10 \) mm, and the coefficient of static friction between the threads and the mating grooves is 0.18. What couple must be exerted on the turnbuckle of member CD to start loosening it?

**Solution:** The complete structure as a free body: The equilibrium conditions:

\[ \sum M_A = -2(2) - 4(4) + 8H = 0, \]
from which \( H = \frac{20}{8} = 2.5 \) kN.

\[ \sum F_y = A_y + H - 2 - 4 = 0, \]
from which \( A_y = 3.5 \) kN.

\[ \sum F_x = A_x = 0. \]

The method of joints: The interior angles \( GF, DF, BD, \) and \( AC \) are each \( \beta = 45^\circ \).

Joint H: (1) \( 0 = -GH \cos \beta - FH, \)
(2) \( 0 = GH \sin \beta + H. \)

Joint F: (3) \( 0 = FH - DF, \)
(4) \( GF = 0. \)

Joint G: (5) \( 0 = GH \cos \beta - EG - DG \cos \beta, \)
(6) \( 0 = -GH \sin \beta - DG \sin \beta. \)

Joint E: (7) \( 0 = EG - CE, \)
(8) \( 0 = DE. \)

Joint D: (9) \( 0 = DG \cos \beta - CD \cos \beta + DF - DB, \)
(10) \( 0 = DG \sin \beta + CD \sin \beta + DE - 4. \)

Joint C: (11) \( 0 = CD \cos \beta - AC \cos \beta + CE, \)
(12) \( 0 = -CD \sin \beta - AC \sin \beta - BC. \)

Joint B: (13) \( 0 = BD - AB, \)
(14) \( 0 = BC - 2. \)

Joint A: (15) \( 0 = AB + A_x + AC \cos \beta, \)
(16) \( 0 = A_x + AC \sin \beta. \)

These equations are solved: The results in detail:

\( GH = -3.54 \) kN,
\( FH = 2.5 \) kN,
\( DF = 2.5 \) kN,
\( GF = 0, \)
\( EG = -5 \) kN,
\( DG = 3.54 \) kN.

\( \alpha = \tan^{-1} \left( \frac{4}{2.5 \times 10} \right) = 3.64^\circ. \)

The static friction angle is \( \alpha_b = \tan^{-1}(0.18) = 10.20^\circ. \) The moment required to loosen the turnbuckle is

\[ M = 2(0.01)(2.12) \tan(10.2^\circ - 3.64^\circ) = 0.00488 \text{ kN-m} \]
\[ m = 4.88 \text{ N-m}. \]
Problem 9.95  In Problem 9.94, what couple must be exerted on the turnbuckle of member \( DG \) to start loosening it?

Solution:  Use the results of the solution of Problem 9.94. The moment required to loosen the turnbuckle is 
\[ M = 2rT \tan(\theta_i - \alpha), \]
where \( r = 0.01 \) m, \( T = DG = 3.54 \) kN, \( \theta_i = 10.2^{\circ} \), and \( \alpha = 3.64^{\circ} \).

\[ M = 2(0.01)(3.54) \tan(10.2^{\circ} - 3.64^{\circ}) = 0.00813 \text{ kN-m} \]

\[ m = 8.13 \text{ N-m} \]
Problem 9.96* The load $W = 800$ N can be raised or lowered by rotating the threaded shaft. The distances are $b = 75$ mm and $h = 200$ mm. The pinned bars are each 300 mm in length. The pitch of the threaded shaft is $p = 5$ mm, the mean radius of the thread is $r = 15$ mm, and the coefficient of kinetic friction between the thread and the mating groove is 0.2. When the system is in the position shown, what couple must be exerted to turn the threaded shaft at a constant rate, raising the load?

Solution: The vertical distances $H_E, B_E, A_D, D_G$ are 100 mm. The included angle $\triangle ABC$ is 
\[ \beta = \sin^{-1} \left( \frac{50 \text{ mm}}{150 \text{ mm}} \right) = 19.47^\circ. \]

The distance $L = 300 \cos \beta$ mm. Isolate the members and write the equilibrium equations, beginning at the top. Isolate the frame $AB$ which supports the load $W$.

\[ \sum M_B = -AL + Wb = 0, \]
\[ \sum F_y = A - W + B_x = 0, \]
\[ \sum F_x = B_x = 0. \]

Isolate $BD$: \[ \sum F_y = -B_x - C_y + D_y = 0, \]
\[ \sum F_x = C_x + D_x = 0. \]
\[ \sum M_D = \left( \frac{h}{2} \right) C_x - \left( \frac{L}{2} \right) C_y - LB = 0 \]

Isolate $DH$: \[ \sum F_y = -D_x - F_x + H_y = 0, \]
\[ \sum F_x = -D_x - F_x + H_x = 0, \]
\[ \sum M_H = \left( \frac{h}{2} \right) F_x + \left( \frac{L}{2} \right) F_y + LD_y \]
\[ + \left( \frac{h}{2} \right) D_x = 0 \]

Isolate $AE$: \[ \sum F_y = -A + C_y + E_y = 0, \]
\[ \sum F_x = C_x + E_x = 0, \]
\[ \sum M_E = - \left( \frac{h}{2} \right) (C_x) - \left( \frac{L}{2} \right) C_y + LA = 0. \]

Isolate $EG$: \[ \sum F_y = -E_x + F_x + G_x = 0, \]
\[ \sum F_x = -E_x + F_x + G_x = 0, \]
\[ \sum M_G = \frac{h}{4} F_x + \left( \frac{L}{2} \right) F_y - LE_y \]
\[ + \left( \frac{h}{2} \right) E_x = 0. \]

These 14 equations in 14 unknowns are to be solved to determine the reaction $G_x$, which is the force that the threaded shaft must overcome to raise the load at a constant rate. An analytic solution is obtained as follows:
Check: This value of $G_y$ is expected from the overall equilibrium conditions. 

Substitute:

$G_y = \frac{1200W}{h} \cos \beta N$. 

Note that

$\cos \beta = \sqrt{1 - \left(\frac{h}{600}\right)^2}$, 

from which

$G_y = 2W \sqrt{500^2 - h^2}$. 

The pitch angle is

$\alpha = \tan^{-1}\left(\frac{5}{2\pi(15)}\right) = 3.037^\circ$. 

The angle of kinetic friction is

$\theta_k = \tan^{-1}(0.2) = 11.31^\circ$. 

The moment required to raise the load at a constant rate:

$M = rG_y \tan(\theta_k + \alpha) = 0.003836G_y = 17.36 \text{ N m}$. 

### Problem 9.97

In Active Example 9.6, suppose that the placement of the winch at $A$ is changed so that the angle between the rope from $A$ to $P$ and the horizontal increases from $45^\circ$ to $60^\circ$. If the suspended load weighs $1500 \text{ N}$, what tension must the winch exert on the rope to raise the load at a constant rate?

Solution: The vector sum of the forces exerted on the pulley by the rope is

$$F = \sqrt{(W + T \sin 60^\circ)^2 + (T \cos 60^\circ)^2}.$$ 

The clockwise couple exerted on the pulley by the rope is

$M = (6 \text{ cm})(T - W)$. 

The angle of kinetic friction is $\theta_k = \tan^{-1}(\mu_k) = \tan^{-1}(0.2) = 11.3^\circ$. 

Applying Eq. (9.12),

$M = rF \sin \theta_k$

$(0.6 \text{ cm})(T - W) = (0.5 \text{ cm}) \sqrt{(W + T \sin 60^\circ)^2 + (T \cos 60^\circ)^2} \sin 11.3^\circ$. 

Setting $W = 1500 \text{ N}$ and solving yields $T = 1550 \text{ N}$. 

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Problem 9.98  The radius of the pulley is 4 cm. The pulley is rigidly attached to the horizontal shaft, which is supported by two journal bearings. The radius of the shaft is 1 in, and the combined weight of the pulley and shaft is 20 N. The coefficients of friction between the shaft and the bearings are $\mu_k = 0.30$ and $\mu_s = 0.28$. Determine the largest weight $W$ that can be suspended as shown without causing the stationary shaft to slip in the bearings.

Solution:

\[ \theta_s = \tan^{-1} (0.3) = 16.70^\circ, \quad d = r \sin \theta_s = 0.287 \text{ cm} \]

\[ \sum M_P: (W + 20 \text{ N})(4 \text{ cm} - d) - (20 \text{ N})(4 \text{ cm}) = 0 \]

Solving we find \[ W = 1.548 \text{ N} \]

Problem 9.99  In Problem 9.98, suppose that the weight $W = 4 \text{ N}$. What couple would have to be applied to the horizontal shaft to raise the weight at a constant rate?

Solution:

\[ \theta_s = \tan^{-1} (0.28) = 15.64^\circ, \quad d = r \sin \theta_s = 0.270 \text{ cm}, \]

$W = 4 \text{ lb}$

\[ \sum M_P: -(20 \text{ N})(4 \text{ cm}) - M + (24 \text{ N})(4 \text{ cm} + d) = 0 \]

Solving we find \[ M = 22.5 \text{ N} \cdot \text{cm} \]
Problem 9.100  The pulley is mounted on a horizontal shaft supported by journal bearings. The coefficient of kinetic friction between the shaft and the bearings is \( \mu_k = 0.3 \). The radius of the shaft is 20 mm, and the radius of the pulley is 150 mm. The mass \( m = 10 \) kg. Neglect the masses of the pulley and shaft. What force \( T \) must be applied to the cable to move the mass upward at a constant rate?

Solution:  The angle of kinetic friction is \( \theta_k = \tan^{-1}(\mu_k) = 16.7^\circ \). The moment required to turn the shaft is \( M = (mg + Tr)\sin\theta_k \). The applied moment is \( M = (T - mg)R \) where \( R \) is the radius of the pulley. Equating and reducing:

\[
T = mg \left( \frac{1 + \frac{r}{R} \sin\theta_k}{1 - \frac{r}{R} \sin\theta_k} \right) = (98.1) \left( \frac{1.0383}{0.9617} \right) = 105.92 \text{ N}
\]

Problem 9.101  In Problem 9.100, what force \( T \) must be applied to the cable to lower the mass at a constant rate?

Solution:  Form the solution to Problem 9.100, \( \theta_k = \tan^{-1}(\mu_k) = 16.7^\circ \); and \( M = (mg + Tr)\sin\theta_k \). The applied moment is \( M = (mg - Tr)R \). Substitute and reduce:

\[
T = mg \left( \frac{1 - \frac{r}{R} \sin\theta_k}{1 + \frac{r}{R} \sin\theta_k} \right) = (98.1) \left( \frac{0.9617}{1.0383} \right) = 90.86 \text{ N}
\]

Problem 9.102  The pulley of 8-cm radius is mounted on a shaft of 1-cm radius. The shaft is supported by two journal bearings. The coefficient of static friction between the bearings and the shaft is \( \mu_s = 0.15 \). Neglect the weights of the pulley and shaft. The 50-N block A rests on the floor. If sand is slowly added to the bucket B, what do the bucket and sand weigh when the shaft slips in the bearings?

Solution:  (See Problem 9.100). The angle of static friction is \( \theta_s = \tan^{-1}(\mu_s) = 8.53^\circ \). The moment required to start rotation for both bearings is \( M = r(B + W)\sin\theta_s \). The applied moment is \( M = (B - W)R \), where \( R \) is the radius of the pulley. Substitute and reduce:

\[
B = W \left( \frac{1 + \frac{r}{R} \sin\theta_s}{1 - \frac{r}{R} \sin\theta_s} \right) = (50) \left( \frac{1.0185}{0.9815} \right) = 51.9 \text{ N}
\]
Problem 9.103  The pulley of 50-mm radius is mounted on a shaft of 10-mm radius. The shaft is supported by two journal bearings. The mass of the block A is 8 kg. Neglect the weights of the pulley and shaft. If a force \( T = 84 \text{ N} \) is necessary to raise the block A at a constant rate, what is the coefficient of kinetic friction between the shaft and the bearings?

Solution: The weight is \( W = mg = 78.5 \text{ N} \). The force on the pulley is \[ F = \sqrt{(W + T\sin\alpha)^2 + (T\cos\alpha)^2}, \]
where \( \alpha = 20^\circ \).

\[ F = \sqrt{107.2^2 + 78.9^2} = 133.13 \text{ N}. \]

The moment required to raise the mass at constant rate for both bearings is \[ M = rF\sin\theta_k = 1.33\sin\theta_k. \]

The applied moment is \( M = (T - W)r = 0.276 \text{ N m}. \) Substitute and reduce:

\[ \sin\theta_k = \frac{(T - W)r}{rF} = \frac{0.276}{1.33} = 0.2073, \]

from which

\[ \theta_k = 11.96^\circ \]

and \( \mu_k = \tan(11.96^\circ) = 0.2119 \)
Problem 9.104  The mass of the suspended object is 4 kg. The pulley has a 100-mm radius and is rigidly attached to a horizontal shaft supported by journal bearings. The radius of the horizontal shaft is 10 mm and the coefficient of kinetic friction between the shaft and the bearings is 0.26. What tension must the person exert on the rope to raise the load at a constant rate?

Solution:

\[ R = 0.1 \text{ m} \]
\[ \mu_k = 0.26 \]
Shaft radius 0.01 m
\[ \mu_k(\text{shaft}) = 0.26 \]
\[ \tan \theta_k = \mu_k \]
\[ \theta_k = 14.57^\circ \]
\[ M_s = rF \sin \theta_k \]
\[ m = 4 \text{ kg} \]

To Find \( F \), we must find the forces acting on the shaft.

\[ \sum F_x: O_x - T \cos 25^\circ = 0 \quad (1) \]
\[ \sum F_y: O_y - T \sin 25^\circ - mg = 0 \quad (2) \]
\[ F = \sqrt{O_x^2 + O_y^2} \quad (3) \]
\[ \sum M: RT - Rmg - M_s = 0 \quad (4) \]
\[ M_s = rF \sin \theta_k \quad (5) \]

Unknowns: \( O_x, O_y, T, M_s, F \)

Solving, we get
\[ T = 40.9 \text{ N} \]
Also,
\[ F = 67.6 \text{ N}, \]
\[ M_s = 0.170 \text{ N\cdotm} \]
\[ O_x = 37.1 \text{ N}, \]
\[ O_y = 56.5 \text{ N} \]
Problem 9.105  In Problem 9.104, what tension must the person exert to lower the load at a constant rate?

Solution: This problem is very much like Problem 9.104 — only the direction of \( M_s \) is changed. The analysis is the same except equation (4), which becomes

\[
RT - Rmg + M_s = 0 \quad (4)
\]

We again have 5 eqns. in 5 unknowns. Solving,

\[
T = 37.6 \text{ N}
\]

Also

\[
F = 64.8 \text{ N}, \quad M_s = 0.163 \text{ N-m}
\]

\[
O_x = 34.1 \text{ N}, \quad O_y = 55.1 \text{ N}
\]

Problem 9.106  The radius of the pulley is 200 mm, and it is mounted on a shaft of 20-mm radius. The coefficient of static friction between the pulley and shaft is \( \mu_s = 0.18 \). If \( F_A = 200 \text{ N} \), what is the largest force \( F_B \) that can be applied without causing the pulley to turn? Neglect the weight of the pulley.

Solution: The magnitude of the force on the shaft supporting the pulley is

\[
F = \sqrt{(F_A + F_B \cos 40^\circ)^2 + (F_B \sin 40^\circ)^2}. \quad (1)
\]

The couple exerted on the pulley by the rope is \( M = (0.2 \text{ m}) (F_B - F_A) \). (2) From Equation (9.12), the largest couple which will not cause the shaft to slip is \( M = r F \sin \theta_s \), where \( r = 0.02 \text{ m} \) and \( \theta_s = \arctan(0.18) \approx 10^\circ \). Substituting Equations (1) and (2) into Equation (3), we obtain

\[
(0.2)(F_B - 200) = (0.02) \sin(10^\circ) \sqrt{(200 + F_B \cos 40^\circ)^2 + (F_B \sin 40^\circ)^2}.
\]

Solving this equation, we obtain \( F_B = 206.8 \text{ N} \).
Problem 9.107  The masses of the boxes are \( m_A = 15 \text{ kg} \) and \( m_B = 60 \text{ kg} \). The coefficient of static friction between boxes \( A \) and \( B \) and between box \( B \) and the inclined surface is 0.12. The pulley has a radius of 60 mm and is mounted on a shaft of 10-mm radius. The coefficient of static friction between the pulley and shaft is 0.16. What is the largest force \( F \) for which the boxes will not slip?

Solution:  Like 9.20, but different tensions.

We have 7 unknowns, 4 equilibrium equations, 2 friction equations and one pulley equation.

\[
\sum F_A = T_1 - F - (147.15 \text{ N}) \sin 20^\circ + f_2 = 0 \\
\sum F_A = N_2 - (147.15 \text{ N}) \cos 20^\circ = 0 \\
\sum F_B = T_2 - (588.6 \text{ N}) \sin 20^\circ - f_1 - f_2 = 0 \\
\sum F_B = N_1 - N_2 - (588.6 \text{ N}) \cos 20^\circ = 0 \\
f_1 = 0.12N_1, \ f_2 = 0.12N_2 \\
\text{For the pulley } \theta = \tan^{-1}(0.16), \ d = 10 \text{ mm} \sin \theta, \\
\text{Thus 60 mm} \ (T_1 - T_2) = (T_1 + T_2) d \Rightarrow F = 283 \text{ N}
\]

Problem 9.108  The two pulleys have a radius of 4 cm and are mounted on shafts of 1-cm radius supported by journal bearings. Neglect the weights of the pulleys and shafts. The tension in the spring is 40 N. The coefficient of kinetic friction between the shafts and the bearings is \( k = 0.3 \). What couple \( M \) is required to turn the left pulley at a constant rate?

Solution:  The angle of kinetic friction is \( \theta = \tan^{-1}(0.3) = 16.7^\circ \).

The load on the bearings is \( F = 40 \text{ N} \). The moment required to turn both pulleys at constant rate is \( M = 2rF \sin \theta \). This is equal to the applied moment,

\[
M_{\text{applied}} = 2rF \sin \theta = (2)(0.01)40 \sin(16.7^\circ) = 0.23 \text{ N-m}
\]

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Problem 9.109  The weights of the boxes are $W_A = 65\text{ N}$ and $W_B = 130\text{ N}$. The coefficient of static friction between boxes $A$ and $B$ and between box $B$ and the floor is $0.12$. The pulley has a radius of $4\text{ cm}$ and is mounted on a shaft of $0.8\text{-cm}$ radius. The coefficient of static friction between the pulley and shaft is $0.16$. What is the largest force $F$ for which the boxes will not slip?

Solution:  Like 9.22 with different tensions

We have 7 unknowns, 4 equilibrium equations, 2 friction equations and one pulley equation.

\[
\sum F_{Ax} = -F + T_1 \cos 20^\circ + f_2 = 0
\]

\[
\sum F_{Ay} = N_2 - T_1 \sin 20^\circ - 65 = 0
\]

\[
\sum F_{Bx} = T_2 - f_1 - f_2 = 0
\]

\[
\sum F_{By} = N_1 - N_2 - 130 = 0
\]

\[f_1 = 0.12N_1, \quad f_2 = 0.12N_2\]

For the pulley we have

\[\theta_i = \tan^{-1}(0.16), \quad d = 0.8 \sin \theta_i\]

The total force on the pulley is

\[F_{\text{pulley}} = \sqrt{(T_1 \cos 30^\circ + T_2)^2 + (T_1 \sin 30^\circ)^2}\]

Thus  \[M = F_{\text{pulley}}d = (T_1 - T_2)4 \text{ cm} \Rightarrow F = 43.4\text{ N}\]
**Problem 9.110** The coefficient of kinetic friction between the 100-kg box and the inclined surface is 0.35. Each pulley has a radius of 100 mm and is mounted on a shaft of 5-mm radius supported by journal bearings. The coefficient of kinetic friction between the shafts and the journal bearings is 0.18. Determine the tension $T$ necessary to pull the box up the surface at a constant rate.

**Solution:**

Working through the pulleys

$$
\theta_k = \tan^{-1}(0.18), \quad d = 5 \text{ mm} \sin \theta_k
$$

$$(T - T_2)100 \text{ mm} = d(T + T_2)
$$

$$(T_2 - T_1)100 \text{ mm} = d(T_2 + T_1)
$$

$T_3 = T_2 + T_3
$$

Now do equilibrium and friction on the box

$$
\sum F_x : T_1 + T = (981 \text{ N}) \sin 60^\circ - f = 0
$$

$$
\sum F_y : N - (981 \text{ N}) \cos 60^\circ = 0
$$

$f = 0.35 \text{ N}$

Solving we find $T = 346 \text{ N}$

---

**Problem 9.111** In Active Example 9.7, suppose that the diameters $D_0 = 3.2 \text{ cm}$ and $D_1 = 1.2 \text{ cm}$ and the angle $\alpha = 72^\circ$. What couple is required to turn the shaft at a constant rate?

**Solution:**

The radii $r_o = 1.75 \text{ cm}$ and $r_i = 0.75 \text{ cm}$.

The required couple is given by Eq. (9.13):

$$
M = \frac{2\mu_k F}{3 \cos \alpha} \left( \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right)
$$

$$
= \frac{2 \times 0.18 \times (200 \text{ N})}{3 \cos 72^\circ} \left( \frac{(1.75 \text{ cm})^3 - (0.75 \text{ cm})^3}{(1.75 \text{ cm})^2 - (0.75 \text{ cm})^2} \right) = 153 \text{ N-cm}.
$$

$M = 153 \text{ N-cm}$
Problem 9.112  The circular flat-ended shaft is pressed into the thrust bearing by an axial load of 600 N. The weight of the shaft is negligible. The coefficients of friction between the end of the shaft and the bearing are \( \mu_x = 0.20 \) and \( \mu_y = 0.15 \). What is the largest couple \( M \) that can be applied to the stationary shaft without causing it to rotate in the bearing?

Solution: The required couple is given by Eq. (9.14) with \( \mu_k \) replaced by \( \mu_s \).

\[
M = \frac{2}{3} \mu_s Fr = \frac{2}{3} (0.2)(100 \text{ N})(0.03 \text{ m}) = 0.4 \text{ N-m}.
\]

\( M = 0.4 \text{ N-m} \)

Problem 9.113  The circular flat-ended shaft is pressed into the thrust bearing by an axial load of 100 N. The weight of the shaft is negligible. The coefficients of friction between the end of the shaft and the bearing are \( \mu_x = 0.20 \) and \( \mu_y = 0.15 \). What couple \( M \) is required to rotate the shaft at a constant rate?

Solution: The required couple is given by Eq. (9.14).

\[
M = \frac{2}{3} \mu_s Fr = \frac{2}{3} (0.15)(100 \text{ N})(0.03 \text{ m}) = 0.3 \text{ N-m}.
\]

\( M = 0.3 \text{ N-m} \)
Problem 9.114  The disk $D$ is rigidly attached to the vertical shaft. The shaft has flat ends supported by thrust bearings. The disk and the shaft together have a mass of 220 kg and the diameter of the shaft is 50 mm. The vertical force exerted on the end of the shaft by the upper thrust bearing is 440 N. The coefficient of kinetic friction between the ends of the shaft and the bearings is 0.25. What couple $M$ is required to rotate the shaft at a constant rate?

Solution: There are two thrust bearings, one at the top and one at the bottom.

$F_U = 440$ N

$m = 220$ kg

$\sum F_y:

F_L - F_U - mg = 0$

$F_L = 2598.2$ N.

The couple necessary to turn $D$ at a constant rate is the sum of the couples for the two bearings.

$M_U = \frac{2}{3}\mu_k F_U r$

$M_L = \frac{2}{3}\mu_k F_L r$

$r = 0.025$ m

$\mu_k = 0.25$

Solving,

$M_U = 1.833$ N-m

$M_L = 10.826$

$M_{TOTAL} = 12.7$ N-m
Problem 9.115  Suppose that the ends of the shaft in Problem 9.114 are supported by thrust bearings of the type shown in Fig. 9.14, where \( r_o = 25 \text{ mm} \), \( r_i = 6 \text{ mm} \), \( \alpha = 45^\circ \), and \( \mu_k = 0.25 \). What couple \( M \) is required to rotate the shaft at a constant rate?

Solution:  There are two thrust bearings, one at the top and one at the bottom.

\[
F_U = 440 \text{ N}
\]

\[
m = 220 \text{ kg}
\]

\[
\sum F_y:
\]

\[
F_L - F_U - mg = 0
\]

\[
F_L = 2598.2 \text{ N}
\]

The couple necessary to turn \( D \) at a constant rate is the sum of the couples for the two bearings.

For the bearings used

\[
m = \frac{2 \mu_k F (r_o^2 - r_i^2)}{3 \cos \alpha (r_o^2 - r_i^2)}
\]

\[
\alpha = 45^\circ, \quad r_o = 0.025 \text{ m}
\]

\[
\mu_k = 0.25, \quad r = 0.006 \text{ m}
\]

Thus,

\[
M_U = \frac{2 \mu_k F_U (r_o^2 - r_i^2)}{3 \cos \alpha (r_o^2 - r_i^2)} = 2.7 \text{ N-m}
\]

\[
M_L = \frac{2 \mu_k F_L (r_o^2 - r_i^2)}{3 \cos \alpha (r_o^2 - r_i^2)} = 16.0 \text{ N-m}
\]

\[
M_{TOTAL} = M_U + M_L = 18.7 \text{ N-m}
\]
Problem 9.116  The shaft is supported by thrust bearings that subject it to an axial load of 800 N. The coefficients of kinetic friction between the shaft and the left and right bearings are 0.20 and 0.26, respectively. What couple is required to rotate the shaft at a constant rate?

Solution: The left bearing: The parameters are

\[
\begin{align*}
  r_o &= 38 \text{ mm}, \\
  r_i &= 0, \\
  \alpha &= 45^\circ, \\
  \mu_k &= 0.2, \\
  F &= 800 \text{ N},
\end{align*}
\]

and \( F = 800 \text{ N} \).

The moment required to sustain a constant rate of rotation is

\[
M_{\text{left}} = \frac{2\mu_k F}{3\cos\alpha} \left(\frac{r_o^2 - r_i^2}{r_o^2 - r_i^2}\right) = 5.73 \text{ N m}.
\]

The right bearing: This is a fl at-end bearing. The parameters are \( \mu_k = 0.26, r = 15 \text{ mm}, \) and \( F = 800 \text{ N} \). The moment required to sustain a constant rate of rotation is

\[
M_{\text{right}} = \frac{2\mu_k Fr}{3} = 2.08 \text{ N m}.
\]

The sum of the moments: \( M = 5.73 + 2.08 = 7.81 \text{ N m} \)

Problem 9.117  A motor is used to rotate a paddle for mixing chemicals. The shaft of the motor is coupled to the paddle using a friction clutch of the type shown in Fig. 9.17. The radius of the disks of the clutch is 120 mm, and the coefficient of static friction between the disks is 0.6. If the motor transmits a maximum torque of 15 N-m to the paddle, what minimum normal force between the plates of the clutch is necessary to prevent slipping?

Solution: The moment necessary to prevent slipping is

\[
M = \frac{2\mu_s Fr}{3} = \frac{2(0.6)(0.12)F}{3} = 15 \text{ N m}.
\]

Solve: \( F = 312.5 \text{ N} \)
Problem 9.118  The thrust bearing is supported by contact of the collar $C$ with a fixed plate. The area of contact is an annulus with an inside diameter $D_1 = 40$ mm and an outside diameter $D_2 = 120$ mm. The coefficient of kinetic friction between the collar and the plate is $\mu_k = 0.3$. The force $F = 400$ N. What couple $M$ is required to rotate the shaft at a constant rate? (See Example 9.8.)

**Solution:** This is a thrust bearing with parameters

$$\mu_k = 0.3,$$

$$\alpha = 0,$$

$$r_o = 60\ \text{mm},$$

$$r_i = 20\ \text{mm},$$

and $F = 400$ N.

The moment required to sustain rotation at a constant rate is

$$M = \frac{2\mu_k F}{3} \left( \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right) = 5.2\ \text{N}\ \text{m}$$

Problem 9.119  An experimental automobile brake design works by pressing the red annular plate against the rotating wheel. If $\mu_k = 0.6$, what force $F$ pressing the plate against the wheel is necessary to exert a couple of 200 N-m on the wheel?

**Solution:** This is a thrust bearing with parameters

$$\mu_k = 0.6,$$

$$\alpha = 0,$$

$$r_o = 90\ \text{mm},$$

$$r_i = 50\ \text{mm},$$

and $M = 200$ N-m.

The moment is

$$M = \frac{2\mu_k F}{3} \left( \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right).$$

Solve:

$$F = \frac{3M}{2\mu_k} \left( \frac{r_o^2 - r_i^2}{r_o^3 - r_i^3} \right) = 4635.8\ \text{N}\ \text{m}$$
**Problem 9.120**  In Problem 9.119, suppose that $\mu_k = 0.65$ and the force pressing the plate against the wheel is $F = 2$ kN.

(a) What couple is exerted on the wheel?

(b) What percentage increase in the couple exerted on the wheel is obtained if the outer radius of the brake is increased from 90 mm to 100 mm?

**Solution:**  Use the results of the solution to Problem 9.119, with parameters $\mu_k = 0.65$, $F = 2$ kN.

(a) The moment is

$$M = \frac{2\mu_k F}{3} \left( \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) = 0.0935 \text{ kN-m} = 93.5 \text{ N-m}$$

(b) The new moment is

$$M = \frac{2\mu_k F}{3} \left( \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) = 0.1011 \text{ kN-m} = 101.1 \text{ N-m}.$$  

The percentage increase is

$$\Delta M\% = \left( \frac{101.1 - 93.5}{93.5} \right) 100 = 8.17\%$$

**Problem 9.121**  The coefficient of static friction between the plates of the car’s clutch is 0.8. If the plates are pressed together with a force $F = 2.60$ kN, what is the maximum torque the clutch will support without slipping?

**Solution:**

$$M = \frac{2\mu_s F}{3 \cos \alpha} \left( \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right)$$

where $\alpha = 90^\circ$, $(\cos \alpha = 1)$

$F = 2600$ N,

$r_o = 0.15$ m

$r_i = 0.075$ m

Solving for $M$,

$M = 243$ N-m
Problem 9.122* The "Morse taper" is used to support the workpiece on a machinist’s lathe. The taper is driven into the spindle and is held in place by friction. If the spindle exerts a uniform pressure $p = 187.5 \text{kPa}$ on the taper and $\mu_S = 0.2$, what couple must be exerted about the axis of the taper to loosen it?

Solution: The outer radius of the taper is $r_o = 0.02 \text{ m}$, and the inner radius is $r_i = 0.0125 \text{ m}$. The angle of the taper is

$$\alpha = 90 - \tan^{-1} \left( \frac{r_o - r_i}{L} \right) = 90 - \tan^{-1} \left( \frac{0.02 - 0.0125}{0.18} \right) = 87.6^\circ.$$

The active area of contact of the taper is the area of a truncated cone:

$$A = \frac{\pi (r_o^2 - r_i^2)}{\cos \alpha} = 0.018394 \text{ m}^2.$$

Check: This expression can be verified using the Pappus-Guldinus Theorem (see Example 7.15) where

$$y = \frac{r_o + r_i}{2},$$

and $L = \frac{r_o - r_i}{\cos \alpha}$.

Problem 9.123 In Active Example 9.9, suppose that the left fixed cylinder is replaced by a pulley. Assume that the tensions in the rope on each side of the pulley are approximately equal. What is the smallest force the woman needs to exert on the rope to support the stationary box?

Solution: The tension in the rope between the pulley and the right cylinder is now equal to the weight $W$. Apply Eq. (9.17) to the right cylinder, assuming that slip in the direction of the force is impending.

$$W = F e^{\mu \beta} \Rightarrow F = W e^{-\mu \beta} = (100 \text{ N}) e^{-0.4(0.12)} = 53.3 \text{ N}$$

Problem 9.124 Suppose that you want to lift a 200 N crate off the ground by using a rope looped over a tree limb as shown. The coefficient of static friction between the rope and the limb is 0.2, and the rope is wound 135° around the limb. What force must you exert to begin lifting the crate?

Solution: The force is given by Eq. (9.17) with $T_1 = 200 \text{ N}$, $\mu_S = 0.2$ and $\beta = (135/180)\pi = 2.36 \text{ rad}$.

$$T_2 = T_1 e^{\mu \beta} = (200 \text{ N}) e^{0.2(2.36)} = 320.4 \text{ N}$$
**Problem 9.125** *Winches* are used on sailboats to help support the forces exerted by the sails on the ropes (*sheets*) holding them in position. The winch shown is a post that will rotate in the clockwise direction (seen from above), but will not rotate in the counterclockwise direction. The sail exerts a tension $T_S = 800 \text{ N}$ on the sheet, which is wrapped two complete turns around the winch. The coefficient of static friction between the sheet and the winch is $\mu_s = 0.2$. What tension $T_C$ must the crew member exert on the sheet to prevent it from slipping on the winch?

**Solution:**

$$T_s = T_s \mu_s \beta$$

$$T_s = 800 \text{ N} \quad \mu_s = 0.2$$

$$\beta = 4\pi$$

Solving,

$$T_s = 64.8 \text{ N}$$

---

**Problem 9.126** The coefficient of kinetic friction between the sheet and the winch in Problem 9.125 is $\mu_k = 0.16$. If the crew member wants to let the sheet slip at a constant rate, releasing the sail, what initial tension $T_C$ must he exert on the sheet as it begins slipping?

**Solution:**

$$T_s = T_s \mu_k \beta$$

$$T_s = 800 \text{ N} \quad \mu_k = 0.16$$

$$\beta = 4\pi$$

Solving

$$T_s = 107.1 \text{ N}$$
Problem 9.127  The box A weights 20 N. The rope is wrapped one and one-fourth turns around the fixed wooden post. The coefficients of friction between the rope and post are $\mu_s = 0.15$ and $\mu_k = 0.12$.

(a) What minimum force does the man need to exert to support the stationary box?
(b) What force would the man have to exert to raise the box at a constant rate?

Solution:

(a) $T = (20 \text{ N}) e^{-0.15 \pi / 2} = 6.16 \text{ N}$

(b) $T = (20 \text{ N}) e^{0.12 \pi / 2} = 51.3 \text{ N}$

Problem 9.128  The weight of the block A is $W$. The disk is supported by a smooth bearing. The coefficient of kinetic friction between the disk and the belt is $\mu_k$. What couple $M$ is necessary to turn the disk at a constant rate?

Solution:  The angle is $\beta = \pi$ radians. The tension in the left belt when the belt is slipping on the disk is $T_{\text{left}} = W e^{\mu_k \beta}$. The tension in the right belt is $T_{\text{right}} = W$. The moment applied to the disk is

$$M = R(T_{\text{left}} - T_{\text{right}}) = R(W e^{\mu_k \beta} - W) = RW(e^{\mu_k \pi} - 1).$$

This is the moment that is required to rotate the disk at a constant rate.

Problem 9.129  The couple required to turn the wheel of the exercise bicycle is adjusted by changing the weight $W$. The coefficient of kinetic friction between the wheel and the belt is $\mu_k$. Assume the wheel turns clockwise.

(a) Show that the couple $M$ required to turn the wheel is $M = W R(1 - e^{-3.4 \mu_k})$.
(b) If $W = 200 \text{ N}$ and $\mu_k = 0.2$, what force will the scale $S$ indicate when the bicycle is in use?

Solution:  Let $\beta$ be the angle in radians of the belt contact with wheel. The tension in the top belt when the belt slips is $T_{\text{upper}} = W e^{-\mu_k \beta}$. The tension in the lower belt is $T_{\text{lower}} = W$. The moment applied to the wheel is

$$M = R(T_{\text{lower}} - T_{\text{upper}}) = R W(1 - e^{-\mu_k \beta}).$$

This is the moment required to turn the wheel at a constant rate. The angle $\beta$ in radians is

$$\beta = \pi + (30 - 15) \left( \frac{\pi}{180} \right) = 3.40 \text{ radians},$$

from which $M = R W(1 - e^{-3.4 \mu_k})$. (b) The upper belt tension is

$$T_{\text{upper}} = 200 e^{-3.4 \mu_k} = 101.3 \text{ N}.$$

This is also the reading of the scale $S$.  

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**Problem 9.130** The box $B$ weighs 50 N. The coefficient of friction between the cable and the fixed round supports are $\mu_k = 0.4$ and $\mu_k = 0.3$.

(a) What is the minimum force $F$ required to support the box?
(b) What force $F$ is required to move the box upward at a constant rate?

**Solution:** The angle of contact between the cable and each round support is $\beta = \frac{\pi}{2}$ radians.

(a) Denote the tension in the horizontal part of the cable by $H$. The tension in $H$ is $H = We^{-2\mu\beta}$. The force $F$ is

$$F = He^{-2\mu\beta} = We^{-2\mu\beta},$$

from which $F = 14.23$ N is the force necessary to hold the box stationary.

(b) As the box is being raised,

$$H = We^{\mu\beta},$$

and

$$F = He^{\mu\beta} = We^{2\mu\beta},$$

from which $F = 128.32$ N.

---

**Problem 9.131** The coefficient of static friction between the 50-N box and the inclined surface is 0.10. The coefficient of static friction between the rope and the fixed cylinder is 0.05. Determine the force the woman must exert on the rope to cause the box to start moving up the inclined surface.

**Solution:** The contact angle between the rope and the fixed cylinder is $\beta = \frac{100 - 45 - 30}{180 - 30} = \frac{\pi}{12}$ radians.

We have 2 equilibrium and 2 friction equations

\[ \sum F_x : T_2 \cos 25° - f - 50 \sin 20° = 0 \]

\[ \sum F_y : T_2 \sin 25° + N - 50 \cos 20° = 0 \]

$f = 0.1$ N

$\sum T : T_2 = 0.05(7\pi/12)$

Solving: $T = 25.2$ N
Problem 9.132 In Problem 9.131, what is the minimum force the woman must exert on the rope to hold the box in equilibrium on the inclined surface?

Solution: See 9.131 - Change the friction force

\[ \sum F = T \cos 25° + f - 50 \sin 20° = 0 \]
\[ \sum F = T \sin 25° + N - 50 \cos 20° = 0 \]
\[ f = 0.1 \text{ N} \]
\[ T = T e^{0.05 \sin 12} \]

Solving \( T = 13.10 \text{ N} \)

Problem 9.133 Blocks B and C each have a mass of 20 kg. The coefficient of static friction at the contacting surfaces is 0.2. Block A is suspended by a rope that passes over a fixed cylinder and is attached to block B. The coefficient of static friction between the rope and the cylinder is 0.3. What is the largest mass block A can have without causing block B to slip to the left?

Solution: First we determine what tension in the rope attached to B will cause B to be on the verge of slipping to the left. Assuming slip of blocks B and C to be impending, the equilibrium equations for block B are

\[ \sum F_x = -T_1 + P \sin 20° + \mu_s P \cos 20° + \mu_s N = 0, \]
\[ \sum F_y = -P \cos 20° + \mu_s P \sin 20° - N - mg = 0. \]

The equilibrium equations for block C are

\[ \sum F_x = -N - P \cos 20° - \mu_s P \sin 20° = 0, \]
\[ \sum F_y = -\mu_s N - P \sin 20° = 0. \]

Substituting the given values and solving yields

\[ T_1 = 220 \text{ N}, N = 420 \text{ N}, P = 256 \text{ N}, R = 136 \text{ N}. \]

Now assume that block A is on the verge of slipping downward. From Eq. (9.17),

\[ T_2 = T_1 e^{\mu_s \beta} = (220 \text{ N}) e^{0.3 \sin 12} = 352 \text{ N}. \]

Thus

\[ m_A = \frac{352 \text{ N}}{9.81 \text{ m/s}^2} = 35.9 \text{ kg} \]

35.9 kg.
Problem 9.134 If the force $F$ in Example 9.10 is increased to 400 N, what are the largest values of the couples $M_A$ and $M_B$ for which the belt will not slip?

Solution: From Example 9.10, $b = 500$ mm, $\mu_s = 0.8$, $R_a = 200$ mm, $R_b = 100$ mm. The angle of contact for pulley $A$ is $\beta_a = \pi + 2\alpha$. The angle of contact for pulley $B$ is $\beta_b = \pi - 2\alpha$, where

$$\alpha = \sin^{-1} \left( \frac{R_a - R_b}{b} \right) = \sin^{-1} \left( \frac{0.1}{0.5} \right) = 0.2014 \text{ radians.}$$

The belt contact is less for pulley $B$, so it is most likely to slip first. The couples are in opposition so that the tension in the upper belt is greater than the tension in the lower belt: For belt $B$:

$$T_{\text{upper}} = T_{\text{lower}} e^{\mu_b b} = 8.945 T_{\text{lower}}.$$

The force is

$$F = (T_{\text{upper}} + T_{\text{lower}}) \cos \alpha,$$

from which

$$T_{\text{lower}} = \frac{F}{(1 + e^{\mu_b b}) \cos \alpha} = 41.05 \text{ N},$$

and $T_{\text{upper}} = 8.945 T_{\text{lower}} = 367.19 \text{ N}$.

The couples are

$$M_b = R_b (T_{\text{upper}} - T_{\text{lower}}) = 32.61 \text{ N-m},$$

$$M_a = R_a (T_{\text{upper}} - T_{\text{lower}}) = 65.23 \text{ N-m}$$
Problem 9.135 The spring exerts a 320-N force on the left pulley. The coefficient of static friction between the flat belt and the pulleys is $\mu_s = 0.5$. The right pulley cannot rotate. What is the largest couple $M$ that can be exerted on the left pulley without causing the belt to slip?

**Solution:** The angle of the belt relative to the horizontal is

$$\alpha = \sin^{-1} \left( \frac{100 - 40}{260} \right) = 0.2329 \text{ radians}.$$ 

For the right pulley the angle of contact is $\theta_{\text{right}} = \alpha - 2\alpha = 2.676 \text{ radians}$. The sum of the horizontal components of the tensions equals the force exerted by the spring:

$$F = (T_{\text{upper}} + T_{\text{lower}}) \cos \alpha = 320 \text{ N}.$$ 

Since the angle of contact is less on the right pulley, it should slip there first. At impending slip, the tensions are related by

$$T_{\text{upper}} = T_{\text{lower}} \mu_s \tan \theta_{\text{right}} = 3.811 T_{\text{lower}}.$$ 

Substitute and solve:

$$T_{\text{lower}} (1 + \mu_s^2 \tan ^2 \theta_{\text{right}}) = \frac{320}{\cos \alpha},$$

from which

$$T_{\text{lower}} = 68.34 \text{ N},$$

and $T_{\text{upper}} = 260.48 \text{ N}.$

The moment applied to the wheel on the right is

$$M_{\text{applied}} = R (T_{\text{upper}} - T_{\text{lower}}) = 0.1 (192.16) = 19.22 \text{ N-m}.$$ 

Problem 9.136 The weight of the box is $W = 30 \text{ N}$, and the force $F$ is perpendicular to the inclined surface. The coefficient of static friction between the box and the inclined surface is $\mu_s = 0.2$.

(a) If $F = 30 \text{ N}$, what is the magnitude of the friction force exerted on the stationary box?

(b) If $F = 10 \text{ N}$, show that the box cannot remain at rest on the inclined surface.

**Solution:** The maximum friction force is defined to be $f = \mu_s N$, where $N$ is the normal force.

(a) The box is stationary, hence the friction force is equal to the force acting to move the box down the plane:

$$\sum F_p = f - W_p = 0,$$

from which $f = W_p = W \sin \alpha = 10.26 \text{ N}$

(b) The component of force parallel to the surface is $W_p = W \sin \alpha = 10.26 \text{ N}$ acting to move the box down the plane. The friction force is $f = \mu_s (10 + 30 \cos \alpha) = 7.638 \text{ N}$, acting to hold the box in place. Since $W_p > f$, the box will move.
Problem 9.137  In Problem 9.136, what is the smallest force \( F \) necessary to hold the box stationary on the inclined surface?

**Solution:** At impending slip, the sum of the forces parallel to the surface is
\[
\sum F_P = f - W_P = 0,
\]
from which \( f = W_P \). The friction force is \( f = \mu_s (F + W \cos \alpha) \), and \( W_P = W \sin \alpha \). Equate and solve:
\[
F = W \left( \frac{\sin \alpha}{\mu_s} - \cos \alpha \right) = 30 \left( \frac{\sin 20^\circ}{0.2} - \cos 20^\circ \right) = 23.1 \text{ N}
\]

Problem 9.138  Blocks \( A \) and \( B \) are connected by a horizontal bar. The coefficient of static friction between the inclined surface and the 400-N block \( A \) is 0.3. The coefficient of static friction between the surface and the 300-N block \( B \) is 0.5. What is the smallest force \( F \) that will prevent the blocks from slipping down the surface?

**Solution:** The (horizontal) connecting bar exerts a component of force normal to the inclined surface. This force increases the normal force exerted on \( B \) by the inclined plane, and reduces the normal force exerted on \( A \) by the inclined plane. Isolate \( B \). Denote the component of the linkage force parallel to the surface by \( f_{\text{link}} \). The equilibrium conditions on \( B \) are
\[
\sum F = -f_{\text{link}} + B \sin \alpha - \mu_{AB} N_B = 0,
\]
where \( N_B = B \cos \alpha + f_{\text{link}} \) (where \( f_{\text{link}} \) forms the sides of a right triangle), from which
\[
f_{\text{link}} = \frac{B}{(1 + \mu_{AB})} (\sin \alpha - \mu_{AB} \cos \alpha) = 70.71 \text{ N}.
\]
Isolate \( A \). The equilibrium conditions are
\[
\sum F = f_{\text{link}} - F + A \sin \alpha - \mu_{AB} N_A = 0,
\]
where \( N_A = A \cos \alpha - f_{\text{link}} \), from which
\[
F - f_{\text{link}} = A (\sin \alpha - \mu_{AB} \cos \alpha) + \mu_{AB} f_{\text{link}} = 219.2 \text{ N}.
\]
The total force required to keep the blocks from slipping is
\[
F = 70.71 + 219.2 = 289.91 \text{ N}
\]
Problem 9.139  What force $F$ is necessary to cause the blocks in Problem 9.138 to start sliding up the plane?

Solution: The friction forces oppose impending motion up the plane. Use the results of the solution to Problem 9.138 with the friction forces reversed. Isolate $B$. The linkage force is

$$f_{\text{link}} = \frac{B}{(1 - \mu_B)}(\sin \alpha + \mu_B \cos \alpha) = 636.4 \text{ N}.$$ 

Isolate $A$. The resultant force on $A$ is

$$F - f_{\text{link}} = A(\sin \alpha + \mu_A \cos \alpha) - \mu_A f_{\text{link}} = 176.78 \text{ N}.$$ 

The resultant force required to cause the blocks to start to move up the plane is $F = 636.4 + 176.8 = 813.2 \text{ N}$

Problem 9.140  The masses of crates $A$ and $B$ are 25 kg and 30 kg, respectively. The coefficient of static friction between the contacting surfaces is $\mu_s = 0.34$. What is the largest value of $\alpha$ for which the crates will remain in equilibrium?

Solution: Choose a coordinate system with the $x$ axis parallel to the inclined surface. Denote the tension in the cables by $T$. Suppose that at impending slip the lower box tends to move up the plane and the upper box tends to move down the plane. Thus, for the lower box:

$$\sum F_x = -W_y \sin \alpha - \mu_s (W_A + W_B) \cos \alpha - \mu_s W_A \cos \alpha + 2T = 0.$$ 

For the upper box,

$$\sum F_x = +\mu_s W_A \cos \alpha - W_A \sin \alpha + T = 0.$$ 

Eliminate $T$ from the two equations, and reduce:

$$(W_y - 2W_A) \sin \alpha + \mu_s (4W_A + W_B) \cos \alpha = 0,$$

from which

$$\alpha = \tan^{-1} \left( \frac{\mu_s (4W_A + W_B)}{(2W_A - W_B)} \right) = \tan^{-1} \left( \frac{0.34(1275.3)}{196.2} \right) = 65.65^\circ.$$
Problem 9.141  The side of a soil embankment has a 45° slope (Fig. a). If the coefficient of static friction of soil on soil is \( \mu_s = 0.6 \), will the embankment be stable or will it collapse? If it will collapse, what is the smallest slope that can be stable?

**Strategy:**  Draw a free-body diagram by isolating part of the embankment as shown in Fig. b.

___

**Solution:**  The strategy is to analyze the free body diagram formed by isolating part of the embankment, as shown.

The sum of the force parallel to the slope are:

\[
\sum F_x = -W \sin \theta + \mu_s W \cos \theta = 0,
\]

from which the required value of the coefficient of static friction is:

\[
\mu_s = \tan \theta = \tan 45° = 1. \]

Since the coefficient of static friction of soil on soil is less than the required value, the embankment will collapse. The smallest slope that will be stable is \( \alpha = \tan^{-1}(0.6) \approx 30.96 = 31° \). This problem is very similar to the problem of the box on an incline.
Problem 9.142 The mass of the van is 2250 kg, and the coefficient of static friction between its tires and the road is 0.6. If its front wheels are locked and its rear wheels can turn freely, what is the largest value of α for which it can remain in equilibrium?

Solution: Choose a coordinate system with the x axis parallel to the incline. The weight of the van is \( W = mg = 22072.5 \text{ N} \). The moment about the point of contact of the rear wheels is

\[
\sum M_y = (3 - 1.2)W \cos \alpha + 1W \sin \alpha - 3N = 0,
\]

from which the normal force at the front wheels is

\[
N = \frac{W(1.8 \cos \alpha + \sin \alpha)}{3}.
\]

The sum of the forces parallel to the inclined surface is

\[
\sum F_x = +\mu_s N - W \sin \alpha = 0.
\]

Combine and reduce:

\[
\frac{1.8}{3} \mu_s \cos \alpha + \frac{\mu_s - 1}{3} \sin \alpha = 0,
\]

from which

\[
\alpha = \tan^{-1} \left( \frac{1.8 \mu_s}{3 - \mu_s} \right) = \tan^{-1}(0.45) = 24.2^\circ.
\]

Problem 9.143 In Problem 9.142, what is the largest value of α for which the van can remain in equilibrium if it points up the slope?

Solution: The sum of the moments about the point of contact of the rear wheels is

\[
\sum M_y = -1.8W \cos \alpha + 1W \sin \alpha + 3N = 0.
\]

The normal force is

\[
N = \frac{W(1.8 \cos \alpha - \sin \alpha)}{3}.
\]

The sum of forces parallel to the incline is

\[
\sum F_x = +\mu_s N - W \sin \alpha = 0.
\]

Combine and reduce:

\[
\frac{1.8}{3} \mu_s \cos \alpha - \left( \frac{\mu_s}{3} + 1 \right) \sin \alpha = 0,
\]

from which

\[
\alpha = \tan^{-1} \left( \frac{1.8 \mu_s}{\mu_s + 3} \right) = 16.7^\circ.
\]
Problem 9.144  The shelf is designed so that it can be placed at any height on the vertical beam. The shelf is supported by friction between the two horizontal cylinders and the vertical beam. The combined weight of the shelf and camera is $W$. If the coefficient of static friction between the vertical beam and the horizontal cylinders is $\mu_s$, what is the minimum distance $b$ necessary for the shelf to stay in place?

Solution:  Take the sum of the moments about the lower cylinder

$$\sum M_{LC} = +bW - bF_{NU} + \mu_s F_{NU} = 0.$$ 

The sum of the forces

$$\sum F_y = F_{NU} - F_{NL} = 0,$$

from which the normal forces at the two cylinders are equal, and

$$F_N = \frac{bW}{(b - \mu_s t)}.$$ 

The force causing slippage is the weight, which is balanced by the friction force:

$$0 = -W + 2\mu_s F_N = -W + \frac{2\mu_s bW}{(b - \mu_s t)} = 0,$$

from which

$$b = \frac{h - \mu_s t}{2\mu_s} = \left(\frac{1}{2}\right) \left(\frac{h}{\mu_s} - t\right).$$
Problem 9.145 The 20-N homogenous object is supported at A and B. The distance \( h = 4 \) cm, friction can be neglected at B, and the coefficient of static friction at A is 0.4. Determine the largest force \( F \) that can be exerted without causing the object to slip.

Solution: Choose a coordinate system with origin at A and the x axis parallel to the floor. Divide the object into a "rectangular" volume and a "triangular" (wedge) volume. The volume of the lower rectangular portion is \( V_1 = \frac{1}{2} \times 6 \times 24 = 24 \) cm\(^3\). The centroidal coordinates are \( x_1 = 2 \) cm, \( y_1 = 3 \) cm. The wedge has a volume \( V_2 = \frac{1}{2} \times \frac{1}{2} \times 2 \times 4 = 4 \) cm\(^3\), and the centroid is at \( x_2 = \frac{4}{3} \) cm, \( y_2 = 6 - \frac{2}{3} \) cm.

The center of mass is located at
\[
x = \frac{2(24) + (4/3)4}{28} = 1.90 \text{ cm}
\]
The moment about B is
\[
\sum M_B = -hF + W(4 - x) - 4A = 0,
\]
from which the normal force at A is
\[
A = \frac{-hF + (4 - x)W}{4}.
\]
The sum of the forces parallel to x is
\[
\sum F_x = -\mu_s A + F = 0,
\]
from which
\[
F(1 + \frac{\mu_s h}{4}) - \frac{\mu_s(4 - x)W}{4} = 0,
\]
and
\[
F = \frac{\mu_s(4 - x)W}{(4 + \mu_s h)} = 2.993 = 2.99 \text{ N}
\]
Problem 9.146  In Problem 9.145, suppose that the coefficient of static friction at B is 0.36. What is the largest value of h for which the object will slip before it tips over?

Solution: Use the solution to Problem 9.145, as applicable. Tipping is imminent when the normal force at A becomes zero. From the solution to Problem 9.145,

\[ A = \frac{-hF + (4 - x)W}{4} \]

from which

\[ h = \frac{(4 - x)W}{F} \]

The sum of the forces parallel to the x axis

\[ \sum F_x = -\mu_s A - \mu_s B + F = 0, \]

from which, for \( A = 0 \), \( F = \mu_s W \). Combine and reduce to obtain

\[ h_{tip} = \frac{(4 - x)}{\mu_s} = 5.82 \text{ cm} \]

Problem 9.147  The 900 N climber is supported in the "chimney" by the normal and friction forces exerted on his shoes and back. The static coefficients of friction between his shoes and the wall and between his back and the wall are 0.8 and 0.6, respectively. What is the minimum normal force his shoes must exert?

Solution: Choose a coordinate system with the x axis horizontal and y axis vertical. Let \( N_s \) be the normal force exerted by shoes, \( N_b \) the normal force exerted by his back. The sum of the forces:

\[ \sum F_x = N_s \cos 4^\circ - N_b \cos 3^\circ - \mu_s N_s \sin 4^\circ + \mu_s N_b \sin 3^\circ = 0, \]

\[ \sum F_y = N_s \sin 4^\circ + N_b \sin 3^\circ + \mu_s N_s \cos 4^\circ + \mu_s N_b \cos 3^\circ - W = 0. \]

Reduce to two simultaneous equations in two unknowns:

\[ a_{11} N_s + a_{12} N_b = 0, \]

and

\[ a_{21} N_s + a_{22} N_b = W, \]

where

\[ a_{11} = \cos 4^\circ - \mu_s \sin 4^\circ = 0.9418, \]

\[ a_{12} = -\cos 3^\circ + \mu_s \sin 3^\circ = -0.9672, \]

\[ a_{21} = \sin 4^\circ + \mu_s \cos 4^\circ = 0.8678, \]

and

\[ a_{22} = \sin 3^\circ + \mu_s \cos 3^\circ = 0.6515. \]

The equations have the solutions

\[ N_s = \frac{a_{12} W}{\det}, \]

and

\[ N_b = \frac{a_{11} W}{\det}, \]

where

\[ \det = a_{11} a_{22} - a_{12} a_{21} = 1.4529 \]

is the determinant of the coefficients. The result: \( N_s = 599 \text{ N} \)

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Problem 9.148  The sides of the 1000 N door fit loosely into grooves in the walls. Cables at A and B raise the door at a constant rate. The coefficient of kinetic friction between the door and the grooves is $\mu_k = 0.3$. What force must the cable A exert to continue raising the door at a constant rate if the cable at B breaks?

Solution: Since the door fits loosely in the grooves, assume that the moment due to the unbalance when cable B breaks causes the door to contact the upper right and lower left corners. Thus the normal force on the sides occur at these corners. The friction forces at the corners oppose movement. The sum of the moments about the lower left corner is

$$\sum M_L = 0.6A - 1.5W + 1.8N_R - 3\mu_kN_R = 0.$$ 

The sum of the forces:

$$\sum F_x = N_L - N_R = 0,$$

from which $N_L = N_R$, and

$$\sum F_y = A - W - \mu_kN_R - \mu_kN_L = 0.$$ 

Combine to obtain the two simultaneous equations:

$$0.6A + (1.8 - 3\mu_k)N_R = 1.5W,$$

and $A - 2\mu_kN_R = W$.

These have the solution:

$$A = \frac{(2\mu_k)(1.5W)}{\text{det}} - \frac{(1.8 - 3\mu_k)W}{\text{det}},$$

where $\text{det} = 1.8 (\mu_k - 1)$ is the determinant of the coefficients. Reducing:

$$A = \frac{-(3\mu_k - (1.8-3\mu_k)W)}{-1.8(1-\mu_k)} = \frac{W}{1-\mu_k} = 1428.6 \text{ N}.$$
**Problem 9.149** The coefficients of static friction between the tires of the 1000-kg tractor and the ground and between the 450-kg crate and the ground are 0.8 and 0.3, respectively. Starting from rest, what torque must the tractor’s engine exert on the rear wheels to cause the crate to move? (The front wheels can turn freely.)

**Solution:** The weight of the crate is \( W = m_g = 4414.5 \text{ N} \). The force required to produce imminent slip of the crate on level ground is \( F_c = \mu_s W = 0.3(4414.5) = 1324.5 \text{ N} \). This is the friction force exerted by the ground on the tires, \( \sum F_x = -F_c + f_{tires} = 0 \).

The friction force is related to the torque about the axle (both wheels) by

\[
\sum M_{axle} = T - 0.8f_{tires} = 0,
\]

from which \( T = 0.8F_c = 1059.5 \text{ N m} \)

---

**Problem 9.150** In Problem 9.149, what is the most massive crate the tractor can cause to move from rest if its engine can exert sufficient torque? What torque is necessary?

**Solution:** The weight of the tractor is \( W_t = m_g = 9810 \text{ N} \). The sum of the moments about the front wheels is

\[
\sum M_F = +0.8W_t + 0.4\mu_s W_c - 2.2 N = 0,
\]

where \( N \) is the normal force on the rear wheels. The sum of the forces at imminent tire slip is

\[
\sum F_x = -\mu_s W_c + \mu_s N = 0,
\]

from which

\[
N = \left( \frac{\mu_s}{\mu_d} \right) W_c.
\]

Substitute into the first equation and reduce:

\[
W_c = \frac{0.8\mu_s W_t}{(2.2\mu_s - 0.4\mu_s\mu_d)} = 11131.9 \text{ N},
\]

from which the mass of the crate is

\[
m_c = \frac{W_c}{g} = 1134.75 = 1134.8 \text{ kg}.
\]

The friction force on the tires is \( f_{tires} = \mu_s W_c = 3339.6 \text{ N} \), from which the torque on the axle (both wheels) is

\[
T = 0.8f_{tires} = 2671.7 \text{ N m}.
\]
Problem 9.151  The mass of the vehicle is 900 kg, it has rear-wheel drive, and the coefficient of static friction between its tires and the surface is 0.65. The coefficient of static friction between the crate and the surface is 0.4. If the vehicle attempts to pull the crate up the incline, what is the largest value of the mass of the crate for which it will slip up the incline before the vehicle’s tires slip?

Solution:  The normal force between the crate and the incline is \( N_c = W_c \cos \theta \), where \( \theta = 20^\circ \). The drawbar force parallel to the incline is \( F_d = -W_c \sin \theta - \mu_s N_c \). For brevity write \( \psi = \sin \theta + \mu_s \cos \theta \). The horizontal component of the drawbar force at the tractor is \( F_{dh} = F_d \cos \theta = -W_c \psi \cos \theta \). The vertical component of the drawbar force at the tractor is \( F_{dv} = F_d \sin \theta = -W_c \psi \sin \theta \). The weight of the tractor is \( W_t = 900 \text{ g} = 8829 \text{ N} \). The sum of the moments about the front wheels is

\[
\sum M = 1W_t - 0.8F_{dh} - 3.7F_{dv} - 2.5 N = 0,
\]

from which

\[
0 = W_t + (0.8 \cos \theta + 3.7 \sin \theta)W_c \psi - 2.5 N = 0.
\]

The sum of the force parallel to the ground is

\[
\sum F_x = F_{dh} + \mu_s N = 0,
\]

from which

\[
N = \frac{W_c \psi \cos \theta}{\mu_s}
\]

Substitute and reduce:

\[
W_c = \frac{\mu_s W_t}{(1.5 - 0.8 \mu_s \cos \theta - 3.7 \mu_s \sin \theta \sin \theta + \mu_s \cos \theta)}
\]

\[
= 7701.1 \text{ N}.
\]

The mass of the crate is

\[
m_c = \frac{W_c}{g} = 785 \text{ kg}
\]
Problem 9.152  Each 1-m bar has a mass of 4 kg. The coefficient of static friction between the bar and the surface at \( B \) is 0.2. If the system is in equilibrium, what is the magnitude of the friction force exerted on the bar at \( B \)?

Solution: The free body diagrams of the bars are as shown. The equilibrium equations are

Left bar:
\[
\sum F_x = C_x + A_x = 0, \\
\sum F_y = C_y + A_y - mg = 0, \\
\sum M_{\text{leftend}} = (1) \cos 45^\circ A_x - (1) \cos 45^\circ A_y - (0.5) \cos 45^\circ mg \\
= 0.
\]

Right bar:
\[
\sum F_x = -A_x + f \cos 30^\circ - N \sin 30^\circ = 0, \\
\sum F_y = -A_y - mg + f \sin 30^\circ + N \cos 30^\circ = 0, \\
\sum M_{\text{rightend}} = (1) \cos 45^\circ A_x + (1) \cos 45^\circ A_y + (0.5) \cos 45^\circ mg \\
= 0.
\]
Solving, we obtain \( N = 43.8 \) N and \( f = 2.63 \) N.

Problem 9.153  In Problem 9.152, what is the minimum coefficient of static friction between the bar and the surface at \( B \) necessary for the system to be in equilibrium?

Solution: From the solution of Problem 9.152, the normal and friction forces are \( N = 43.8 \) N and \( f = 2.63 \) N. Slip impends when \( f = \mu s N \) so, \( \mu s = \frac{2.63}{43.8} = 0.06. \)
Problem 9.154  The collars \( A \) and \( B \) each have a mass of 2 kg. If friction between collar \( B \) and the bar can be neglected, what minimum coefficient of static friction between collar \( A \) and the bar is necessary for the collars to remain in equilibrium in the position shown?

Solution: The weight of each collar is \( W = mg = 19.62 \) N. Denote \( \theta = 45^\circ, \alpha = 20^\circ \). The unit vector parallel to the bar holding \( B \) is

\[
e_B = -\sin \theta + j \cos \theta.
\]

The weight of \( B \) is \( W = -j |W| \). The components of the weight of \( B \) parallel to the bar is

\[
W_{PB} = (e_B \cdot W) e_B = -|W| \cos \theta e_B.
\]

This force must be balanced by a component of the force in the connecting wire for \( B \) to remain stationary. The components of the tension in the wire are

\[
T = |T|(i \cos(180^\circ + \alpha) + j \sin(180^\circ + \alpha))
\]

\[
= |T|(-\cos \alpha - j \sin \alpha),
\]

from which the component of \( T \) parallel to the bar supporting \( B \) is

\[
T_{PB} = (e_B \cdot T) e_B = |T| \sin(\theta - \alpha) e_B.
\]

The sum of forces along \( e_B \):

\[
|T| \sin(\theta - \alpha) \sin \theta = -|W| \cos \theta \sin \theta = 0,
\]

from which:

\[
\frac{|W|}{|T|} = \frac{\sin(\theta - \alpha)}{\cos \theta}.
\]

The unit vector parallel to the bar supporting \( A \) is \( e_A = j \). The component of \( T \) parallel to the bar supporting \( A \) is \( |T| |N_A| = -j \cdot T = |T| \sin \alpha \), and the force exerted by \( T \) on the slider \( A \) perpendicular to the bar is \( |N_A| = -i \cdot |T| = |T| \cos \alpha \), where the negative sign is used because the tension at \( A \) is in opposition to the tension at \( B \) (tension is reversed).

The sum of forces parallel to the bar is

\[
\sum F_A = -\mu_s |N_A| - |W| + |T_{PA}| = 0,
\]

from which

\[
\mu_s |T| \cos \alpha - |W| + |T| \sin \alpha = 0,
\]

and

\[
\mu_s = \frac{|W|}{|T|} \left( \frac{1}{\cos \alpha} \right) - \tan \alpha.
\]

Substitute and reduce:

\[
\mu_s = \frac{\sin(\theta - \alpha)}{\cos \alpha \cos \theta} - \tan \alpha = 0.272
\]
**Problem 9.155** In Problem 9.154, if the coefficient of static friction has the same value \( \mu_s \) between collars A and B and the bars, what minimum value of \( \mu_s \) is necessary for the collars to remain in equilibrium in the position shown? (Assume that slip impends at A and B.)

**Solution:** The weight of each collar is \( W = mg = 19.62 \text{ N} \). Denote \( \theta = 45^\circ, \alpha = 20^\circ \). Isolate Collar A. The sum of forces:

\[
\sum F_x = T \cos \alpha - N_A = 0.
\]

\[
\sum F_y = T \sin \alpha + \mu_s N_A - mg = 0.
\]

Isolate Collar B: The sum of forces:

\[
\sum F_x = N_B \cos \theta + \mu_s N_B \cos \theta - T \cos \alpha = 0.
\]

\[
\sum F_y = N_B \sin \theta - \mu_s N_B \sin \theta - T \sin \alpha = 0.
\]

These are four equations in four unknowns. Solve: \( T = 45.4 \text{ N}, N_A = 42.6 \text{ N}, N_B = 55 \text{ N}, \) and \( \mu_s = 0.0963 \). This is the minimum coefficient of friction required to maintain equilibrium.

**Problem 9.156** The clamp presses two pieces of wood together. The pitch of the threads is \( p = 2 \text{ mm} \), the mean radius of the thread is \( r = 8 \text{ mm} \), and the coefficient of kinetic friction between the thread and the mating groove is 0.24. What couple must be exerted on the thread shaft to press the pieces of wood together with a force of 200 N?

**Solution:** The free-body diagram of the upper arm of the clamp is shown. From the equilibrium equation

\[
\sum M(\text{BE}) = -(0.25)(200) - 0.1BE = 0.
\]

we find that \( BE = -500 \text{ N} \). The compressive load in \( BE \) is 500 N.

The slope of the thread is

\[
\alpha = \arctan \left( \frac{p}{2\pi r} \right)
\]

\[
= \arctan \left( \frac{0.002}{2\pi(0.008)} \right)
\]

\[
= 2.279^\circ.
\]

The angle of friction is

\[
\theta_k = \arctan(0.24) = 13.496^\circ.
\]

From Eq. (9.9) with \( \theta_k = \theta_k \), the required couple is

\[
M = rF \tan(\theta_k + \alpha)
\]

\[
= (0.008)(500) \tan(13.496^\circ + 2.279^\circ)
\]

\[
= 1.13 \text{ N-m}.
\]
Problem 9.157  In Problem 9.156, the coefficient of static friction between the thread and the mating groove is 0.28. After the threaded shaft is rotated sufficiently to press the pieces of wood together with a force of 200 N, what couple must be exerted on the shaft to loosen it?

Solution: First, find the forces in the parts of the clamp. Then analyze the threaded shaft. BE is a two force member

\[ \sum F_x: \quad BE + C_x = 0 \quad (1) \]

\[ \sum F_y: \quad 200 + C_y = 0 \quad (2) \]

\[ \sum M_A: \quad -0.05BE + 0.05C_x + 0.25C_y = 0 \quad (3) \]

Solving, we get

\[ BE = -500 \text{ N} \quad \text{(compression)} \]

\[ C_x = 500 \text{ N} \]

\[ C_y = -200 \text{ N} \]

We don't have to solve for additional forces because we used the fact that member BE was a two force member.

From Problem 9.170, \( P = 2 \text{ mm, } r = 8 \text{ mm} \). We have \( \mu_k = 0.28 \). We want to loosen the clamp (Turn the clamp such that the motion is in the direction of the axial force).

To do this,

\[ M = rF \tan(\theta_l - \alpha) \]

where

\[ \tan \theta_l = \mu_k = 0.28 \]

\[ \theta_l = 15.64^\circ \]

\[ \tan \frac{\alpha}{2} = \frac{2}{\pi} \]

\[ \alpha = 2.28^\circ \]

\[ F = |BE| = 500 \text{ N} \]

Solving

\[ M = 0.950 \text{ N\cdotm} \]

Problem 9.158  The axles of the tram are supported by journal bearings. The radius of the wheels is 75 mm, the radius of the axles is 15 mm, and the coefficient of kinetic friction between the axles and the bearings is \( \mu_k = 0.14 \). The mass of the tram and its load is 160 kg. If the weight of the tram and its load is evenly divided between the axles, what force \( P \) is necessary to push the tram at a constant speed?

Solution: Assume that there are two bearings per axle. The weight of the tram is \( W = mg = 1569.6 \text{ N} \). This load is divided between four bearings:

\[ F = \frac{W}{4} = 392.4 \text{ N} \]

The angle of kinetic friction is \( \theta_l = \tan^{-1}(\mu_k) = 7.97^\circ \). The moment required to turn each bearing at a constant rate is \( M = Fr \sin \theta_l = 0.8161 \text{ N\cdotm} \), and the force per wheel is

\[ P_w = \frac{M}{R} = \frac{0.8161}{0.075} = 10.88 \text{ N} \]

The total force required to push the tram is

\[ P = 4P_w = 43.5 \text{ N} \]
**Problem 9.159**  The two pulleys have a radius of 120 mm and are mounted on shafts of 20 mm radius supported by journal bearings. Neglect the weights of the pulleys and shafts. The coefficient of kinetic friction between the shafts and the bearings is $\mu_k = 0.2$. If a force $T = 1000 \text{ N}$ is required to raise the man at a constant rate, what is his weight?

**Solution:**  Denote the tension in the horizontal portion of the cable by $H$. The angle of kinetic friction is $\theta_k = \tan^{-1}(0.2) = 11.31^\circ$.

Consider the right pulley: The force on the right pulley is

$$F = \sqrt{T^2 + H^2}.$$  

The magnitude of the moment required to turn the shaft in the bearing is $M_{right} = r_H \sqrt{T^2 + H^2} \sin \theta_k$. The applied moment is $M_{applied} = (T - H) r$, from which $(T - H) r = r_H \sqrt{T^2 + H^2} \sin \theta_k$. Square both sides and reduce to obtain the quadratic:

$$H^2 - 2TH \left( \frac{1}{1 - \left( \frac{H}{r} \right) \sin^2 \theta_k} \right) + T^2 = 0,$$

or $H^2 - 2 \times 1001.07 H + 1 \times 10^6 = 0$.  

This has the solutions: $H = \frac{2002.14 \pm \sqrt{2002.14^2 - 4 \times 10^6}}{2} = 1047.3$,  

954.8 N. The lesser root corresponds to the horizontal tension, $H = 954.8 \text{ N}$.

Consider the left pulley: The force on the pulley is $F_{left} = \sqrt{W^2 + H^2}$.  

The applied moment is $M_{applied} = -(T - W) r$, from which $(T - W) r = r_H \sqrt{W^2 + H^2} \sin \theta_k$. Square both sides and reduce to the quadratic:

$$W^2 - 2W \left( \frac{H}{r} \right) \sin^2 \theta_k W + H^2 = 0,$$

or $W^2 - 2(955.82)W + 911643 = 0$.  

This has the solutions: $W_{1,2} = 955.82 \pm \sqrt{955.82^2 - 911643} = 999.97 \text{ N}, 911.67 \text{ N}$. By an analogous argument to that used in Problem 10.92, the lesser root corresponds to the weight of the man, $W_{\text{raised}} = 911.7 \text{ N}$.
**Problem 9.160** If the man in Problem 9.159 weighs 800 N, what force \( T \) is necessary to lower him at a constant rate?

**Solution:** Use the solution to Problem 9.160, with \( W = 800 \) N.

Begin with the left pulley: the quadratic relation between the weight and the horizontal tension is

\[
H^2 - 2 \left( \frac{W}{1 - \left( \frac{L}{R} \right)^2 \sin^2 \theta_0} \right) H + W^2 = 0,
\]

or \( H^2 - 2(800.856)H + 640000 = 0 \).

This has the solutions:

\[
H_{1,2} = 800.856 \pm \sqrt{800.856^2 - 640000} \approx 837.87 \text{ N, 763.84 N}.
\]

The lesser root corresponds to the horizontal tension: \( H = 763.84 \) kN.

Consider the right pulley: The quadratic relation between the tension and the horizontal tension is

\[
T^2 - 2 \left( \frac{H}{1 - \left( \frac{L}{R} \right)^2 \sin^2 \theta_0} \right) T + H^2 = 0,
\]

or \( T^2 - 2(764.617)T + 583390 = 0 \).

This has the solutions:

\[
T_{1,2} = 764.617 \pm \sqrt{764.617^2 - 583390} \approx 800 \text{ N, 729.27 N}.
\]

By previous arguments, the lesser root corresponds to the tension when the man is being lowered at a constant rate, \( T_{\text{lower}} = 729.3 \) N.
Problem 9.161  If the two cylinders are held fixed, what is the range of $W$ for which the two weights will remain stationary?

Solution:  Denote the tension in the horizontal part of the rope by $H$. The angle of contact with each cylinder is

$$\beta = \frac{\pi}{2}$$

Begin on the left with the known weight. Suppose that the known weight is on the verge (imminent slip) of being lowered. The tension is $T_L = 100\ N$. The tension in the horizontal portion of the rope is

$$H = T_L e^{-0.34\beta} = 58.62\ N.$$  The tension in the right part of the rope is

$$T_R = H e^{-0.30\beta} = 36.59\ N.$$  This is the minimum weight for which the system will remain stationary. Suppose that the weight on the left is on the verge of being raised. The horizontal tension is $H = T_L e^{0.34\beta} = 170.59\ N$. The tension on the right is $T_R = H e^{0.30\beta} = 273.3\ N$. This is the maximum weight for which the system will remain stationary. Thus $36.59 \leq W \leq 273.3\ (N)$

Problem 9.162  In Problem 9.161, if the system is initially stationary and the left cylinder is slowly rotated, determine the largest weight $W$ that can be (a) raised; (b) lowered.

Solution:  Assume that the rope does not slip on the slowly rotating cylinder, but is always at the point of imminent slip. The tension in the horizontal part as the cylinder is rotated is $H = T_L e^{0.34\beta} = 170.6\ N$, where the static coefficient of friction is used, since the rope does not slip. The tension in the right portion of the rope is $T_R = H e^{-0.28\beta} = 109.88 = 110\ N$, where the kinetic coefficient of friction is used, since the rope slips at a steady rate. Thus (a) $W = 110\ N$

(b) If the weight $W$ is slowly being lowered, the tension in the horizontal portion is also $H = T_L e^{0.34\beta} = 170.6\ N$, and the tension in the right portion of the rope is $T_R = H e^{-0.28\beta} = 264.8\ N$, from which $W = 264.8\ N$