1. The damped motion of a vibrating particle is defined by the position vector 
\[ \mathbf{r} = 30 \left(1 - \frac{1}{t+1}\right) \mathbf{i} + 20e^{-\pi t/2} \cos 2\pi t \mathbf{j} \] in millimetres, where \( t \) is expressed in seconds. Determine the position, the velocity and the acceleration of the particle when

(a) \( t = 0 \), and \( \text{Ans.} \ \mathbf{r}(0) = 20 \mathbf{j} \) mm, \( \mathbf{v}(0) = 30 \mathbf{i} - 31.4 \mathbf{j} \) mm/s, \( \mathbf{a}(0) = -60 \mathbf{i} - 740.22 \mathbf{j} \) mm/s²

(b) \( t = 1.5 \) s. \( \text{Ans.} \ \mathbf{r}(1.5) = 18 \mathbf{i} + 1.9 \mathbf{j} \) mm, \( \mathbf{v}(1.5) = 4.8 \mathbf{i} + 3 \mathbf{j} \) mm/s, \( \mathbf{a}(1.5) = -3.84 \mathbf{i} + 70.2 \mathbf{j} \) mm/s²

2. Show that the acceleration of a particle moving with constant speed is either zero or perpendicular to its velocity. (Hint: \( |\mathbf{v}|^2 = \mathbf{v} \cdot \mathbf{v} \))

3. A rocket is released at a point from a jet aircraft flying horizontally at 1000 km/h at an altitude of 800 m. If the rocket thrust remains horizontal and gives the rocket a horizontal acceleration of 0.5\( g \), determine the angle from the horizontal to the line of sight to the target, i.e. the straight line from the point of release of the missile to the target. (Ans. 11.46°)

4. A particle P is projected from the origin with initial speed 5 m/s and projection angle \( \theta \). At time \( t = 0 \), when P is at the origin, another particle Q begins to move on the (horizontal) \( x \) axis with initial speed 15/4 m/s and constant acceleration \( a = 3g/4 \) m/s². Determine the time \( t \) when P collides with Q, and the distance from the origin where this occurs. (Ans. 0.29 s, 1.37 m)

5. A projectile is fired with a speed \( u \) at the entrance to a horizontal tunnel of length 24 m and height 2 m. Determine the minimum value of \( u \) and the maximum projection angle \( \theta \) for which the projectile will reach the other end of the tunnel without touching the top of the tunnel. (Ans. \( u = 19.81 \) m/s, \( \theta = 18.43° \))

6. The diagram shows a projectile fired with initial speed \( v_0 \) and elevation \( \alpha \) (measured from the horizontal) on a sloping terrain inclined at an angle \( \theta \) to the horizontal. Determine the range of the projectile (i.e. calculate the distance \( D \)).

(Ans. \( \frac{2v_0^2}{g \cos \theta} (\sin \alpha \cos \alpha - \cos^2 \alpha \tan \theta) \))

7. The rotor of a jet engine is rotating at 10 000 rpm when the fuel is shut off. Suppose that the subsequent acceleration of the rotor is given by \( \alpha = -0.0005\omega^2 \) (in rad/s²), where \( \omega \)
is the angular velocity in rad/s. Determine how long it takes for the rotor to slow down from 10 000 rpm to 1 000 rpm and the number of revolutions the rotor turns in this time. (Ans. 17.19 s, 733 revs)

8. The angle \( \theta \) measures the direction of the unit vector \( \mathbf{e} \) relative to the \( x \) axis. The angle \( \theta \) is given as a function of time by \( \theta = 2t^2 \) rad. What is the vector \( \frac{d\mathbf{e}}{dt} \) when \( \theta = \frac{\pi}{4} \) rad? (Ans. \( -\sqrt{2} \mathbf{i} + \sqrt{2} \mathbf{j} \))